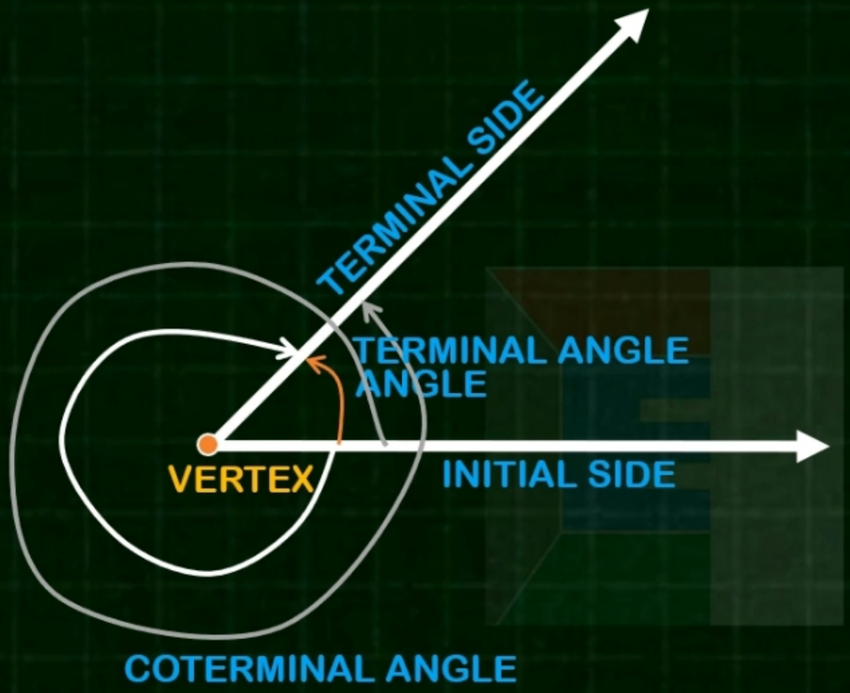
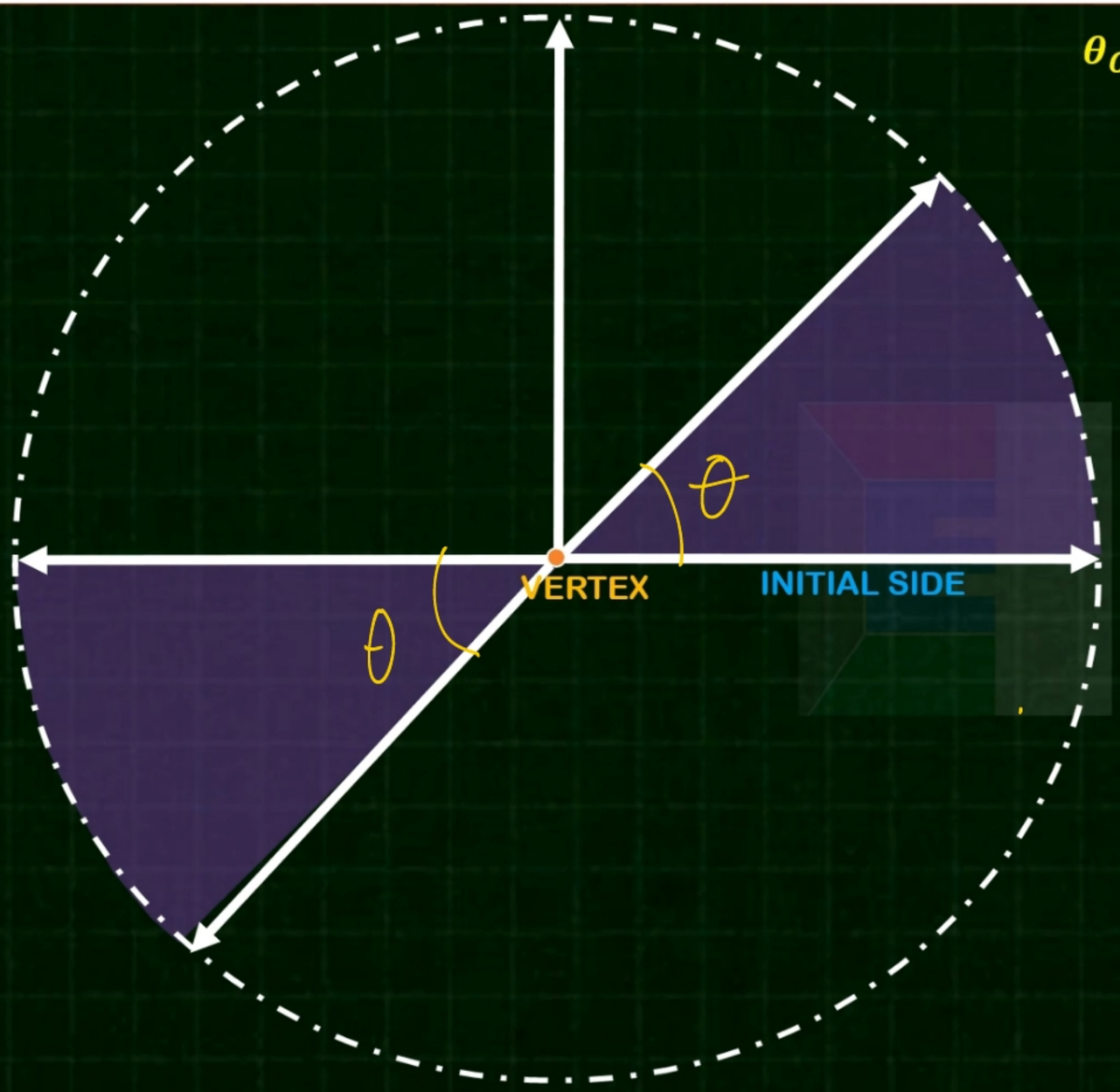
PLANE TRIGONOMETRY

ENGR. DARIO A. CALAQUE, JR.

ANGLES

$$\theta_{COTERMINAL} = \theta_{TERMINAL} \pm n(360)$$





$\theta_{COTERMINAL} = \theta_{TERMINAL} \pm n(360)$

STRAIGHT ANGLE = 180^0

FULL ANGLE = 360^0

ACUTE ANGLE < 90^0

RIGHT ANGLE = 90^0

OBTUSE ANGLE > 90^0

REFLEX ANGLE > 180^0 but < 360^0

COMPLEMENTARY ANGLES are two angles whose SUM is a right angle (90^0)

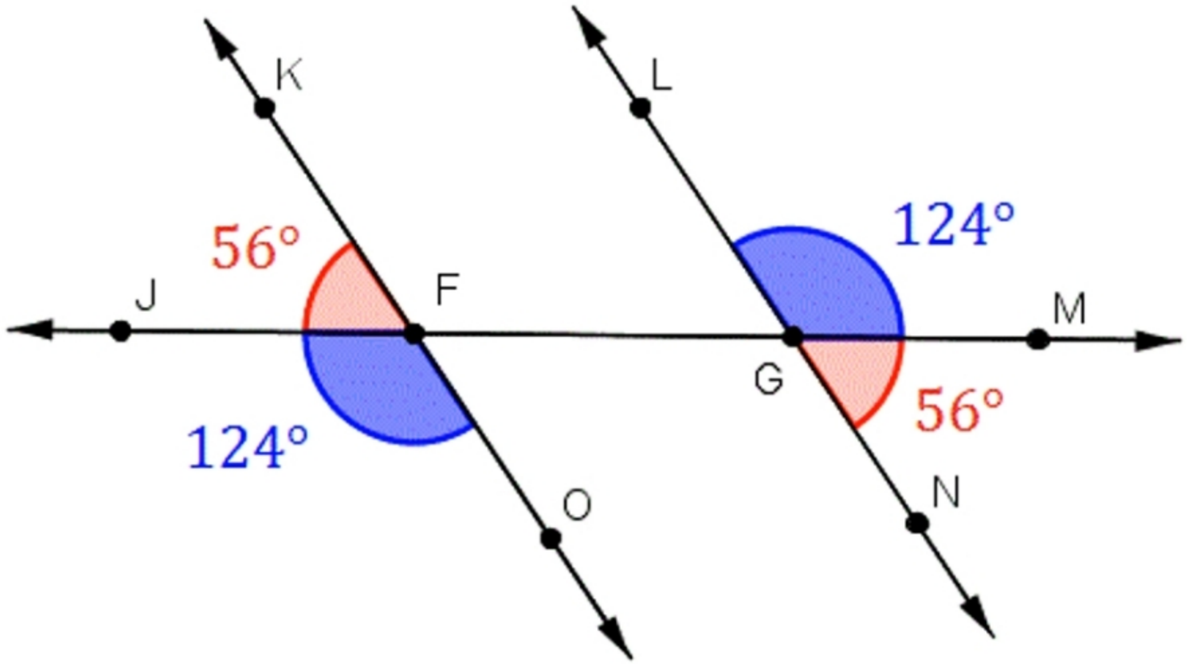
SUPPLEMENTARY ANGLES are two angles whose SUM is a straight angle (180^0)

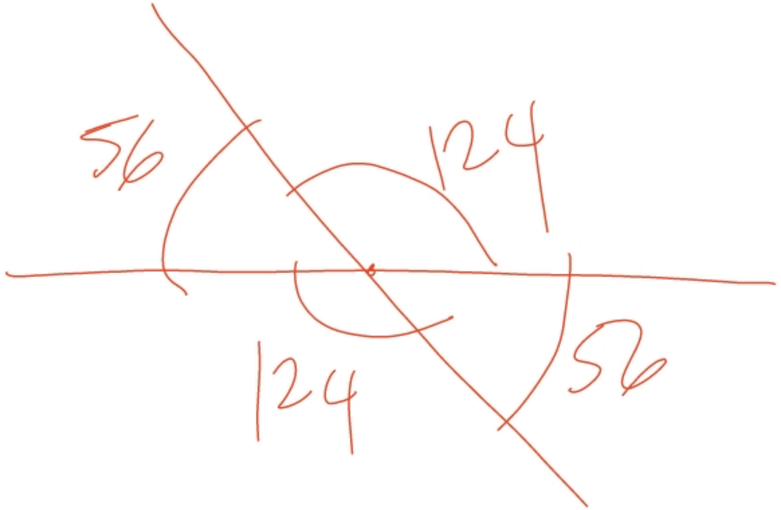
ADJACENT ANGLES are two angles with common arm and vertex

VERTICAL ANGLES are two angles intersect at each other.

ANGLE THEOREMS

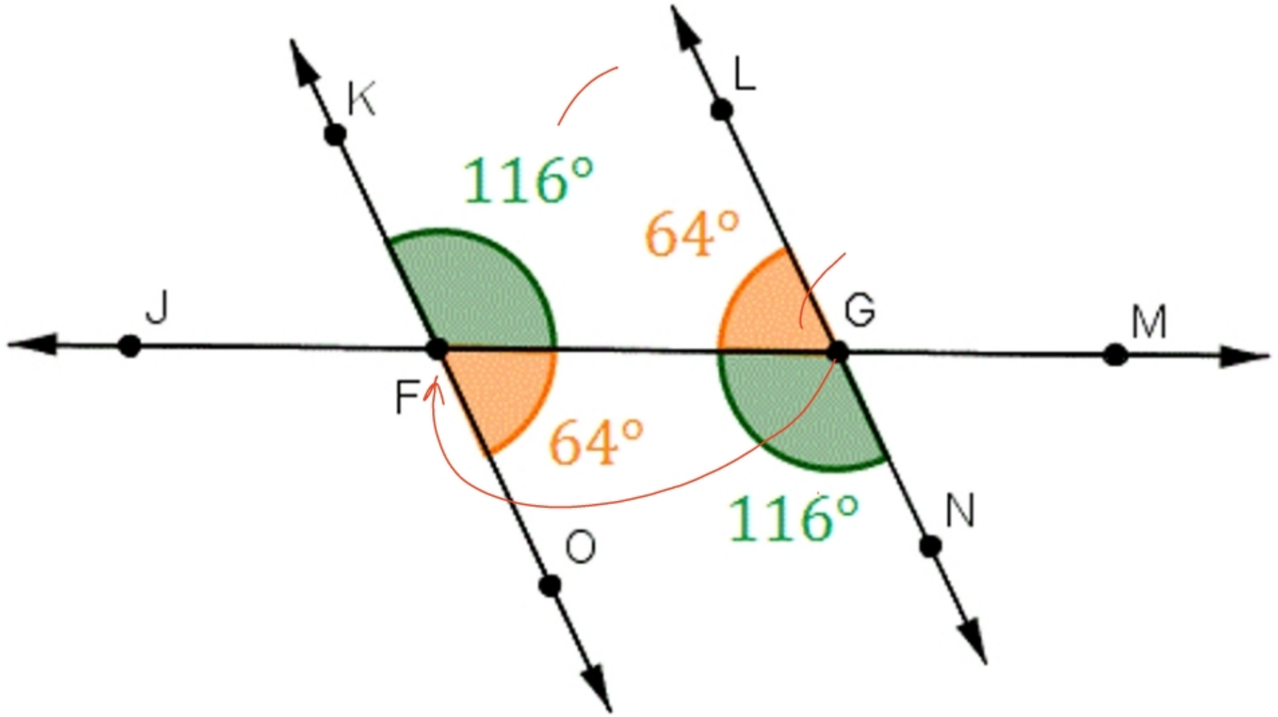
Alternate Exterior Angles Theorem

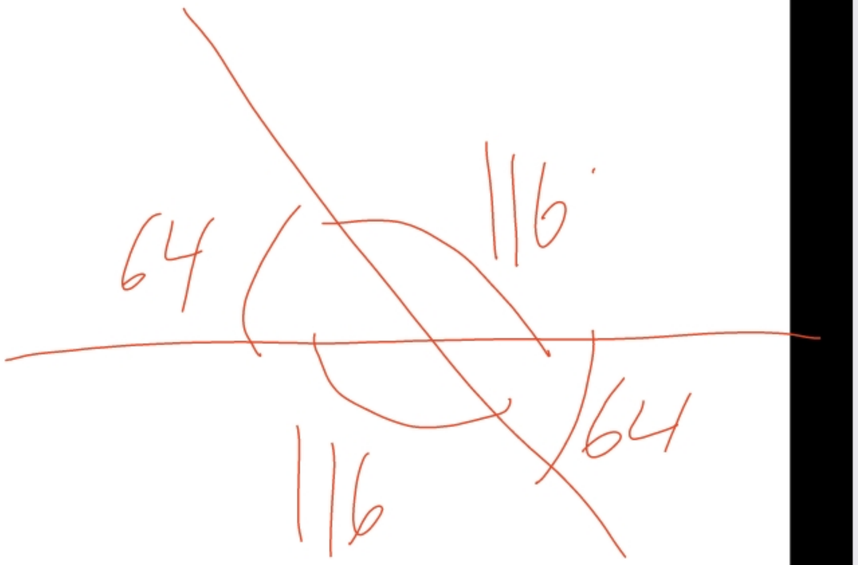




The alternate exterior angles have the same degree measures because the lines are parallel to each other

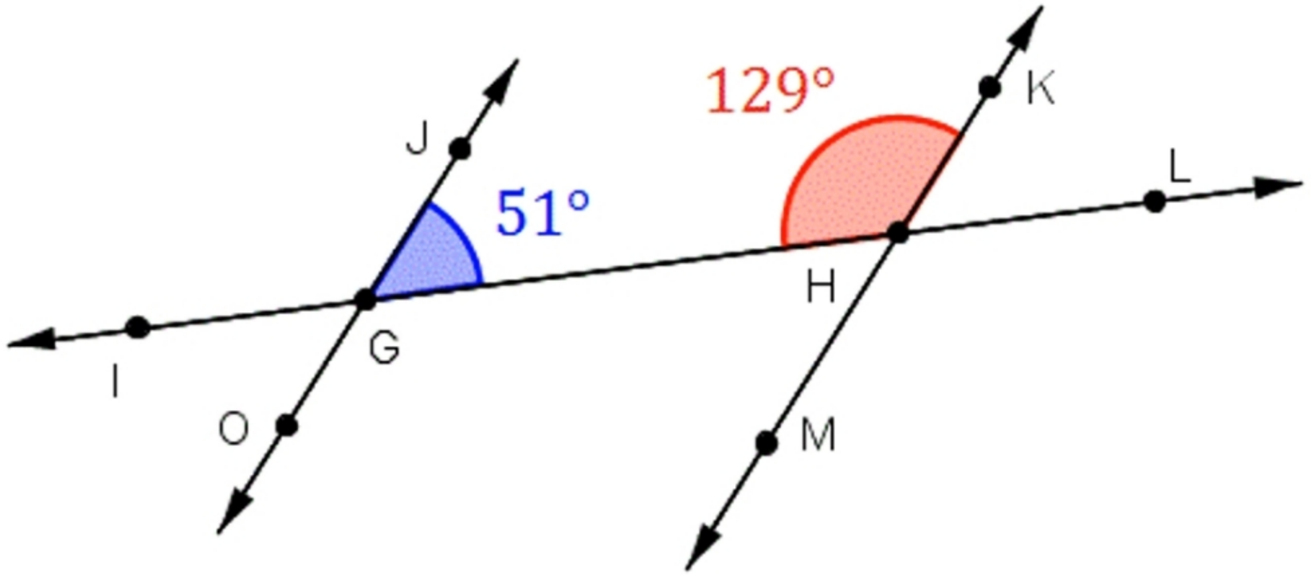
Alternate Interior Angles Theorem





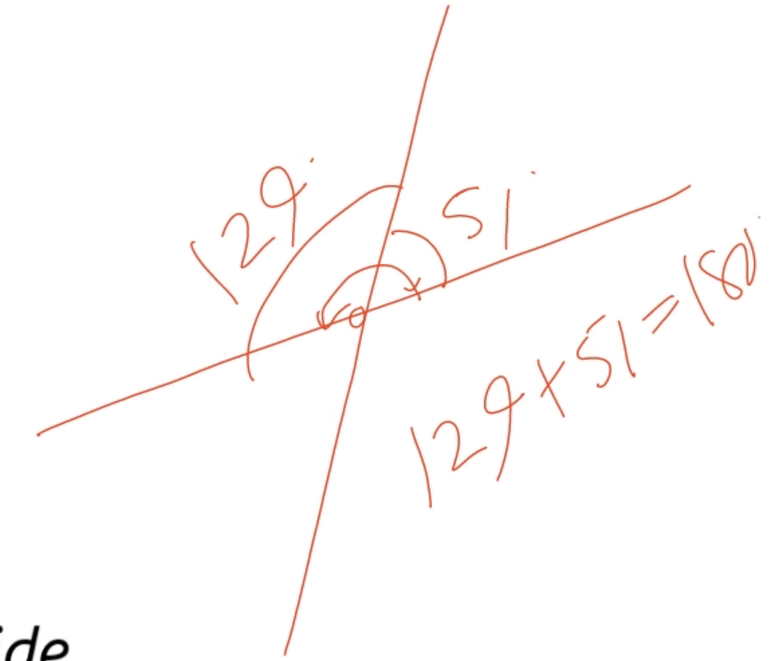
The alternate interior angles have the same degree measures because the lines are parallel to each other.

Same-Side Interior Angles Theorem

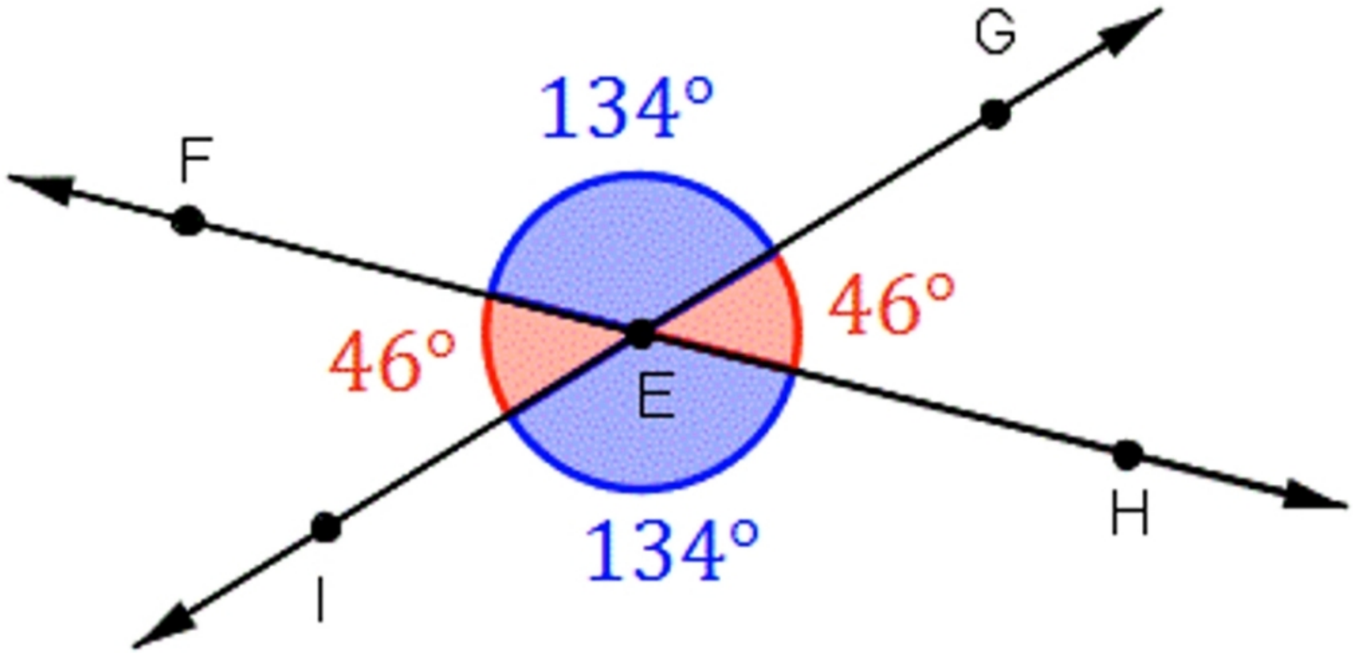


$$51° + 129° = 180°$$

The sum of the degree measures of the same-side interior angles is 180°.

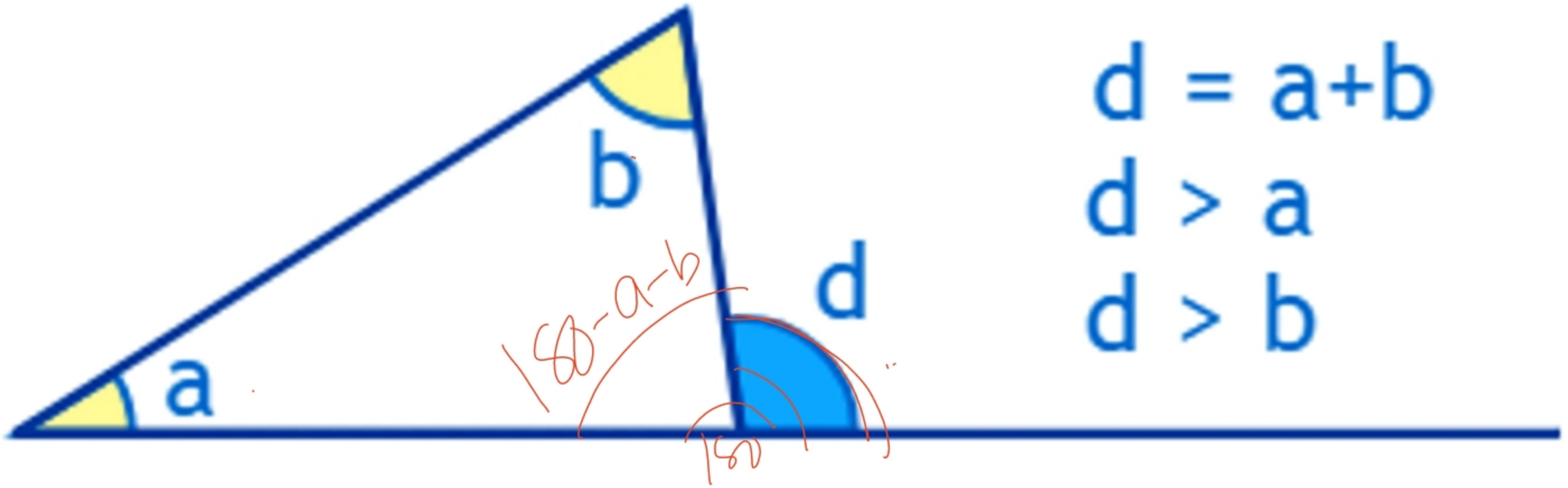


Vertical Angles Theorem



The vertical angles have equal degree measures. There are two pairs of vertical angles.

Exterior Angles Theorem



$d = a+b$

$d > a$

$d > b$

The exterior angle of a triangle equals the sum of the opposite two interior angles

PAST CE BOARD EXAM MAY 2019:

Given that sin34 = 0.5592, find an acute angle such that cosθ = 0.5592

SOLUTION:

$$\cos\theta = 0.5592$$

$$\theta = 56^0$$

PAST CE BOARD EXAM NOV.2017:

FIND THE TWO POSITIVE AND TWO NEGATIVE ANGLES COTERMINAL WITH 10DEGREES.

SOLUTION:

$n = 3$

$$\theta = 10^0 + n(360)$$
$$\theta = 10^0 - n(360)$$

$When\ n = 1$
$$\theta = 10^0 + 1(360) = 370$$
$$\theta = 10^0 - 1(360) = -350$$

$When\ n = 2$
$$\theta = 10^0 + 2(360) = 730$$
$$\theta = 10^0 - 2(360) = -710$$

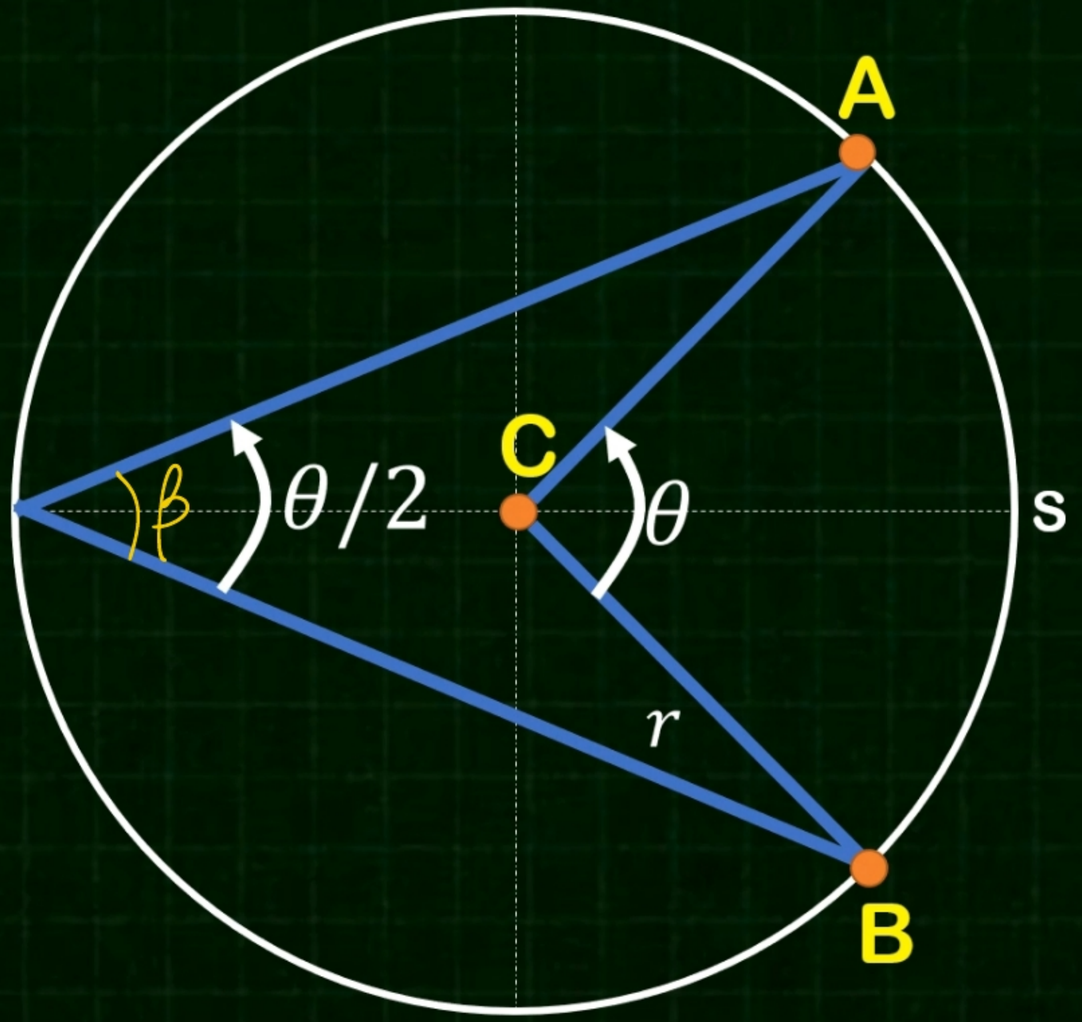
ans.

TWO POSITIVE $\measuredangle$'s $= \begin{cases} 370^\circ \\ 730^\circ \end{cases}$

TWO NEGATIVE $\measuredangle$'s $= \begin{cases} -350^\circ \\ -710^\circ \end{cases}$

CIRCLE THEOREMS

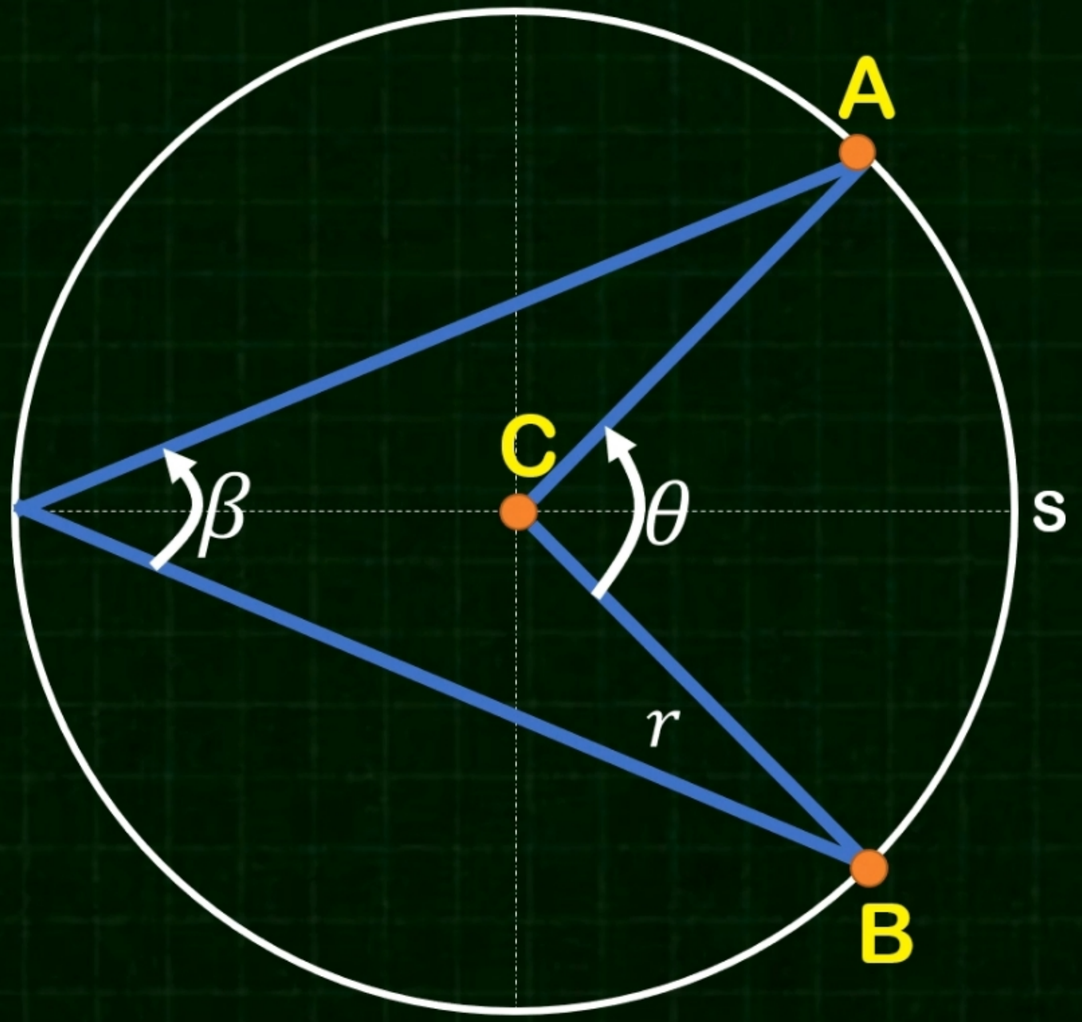
CIRCLE



Peripheral Angle Theorem

$$\beta = \frac{\theta}{2} \quad \text{or} \quad \theta = 2\beta$$

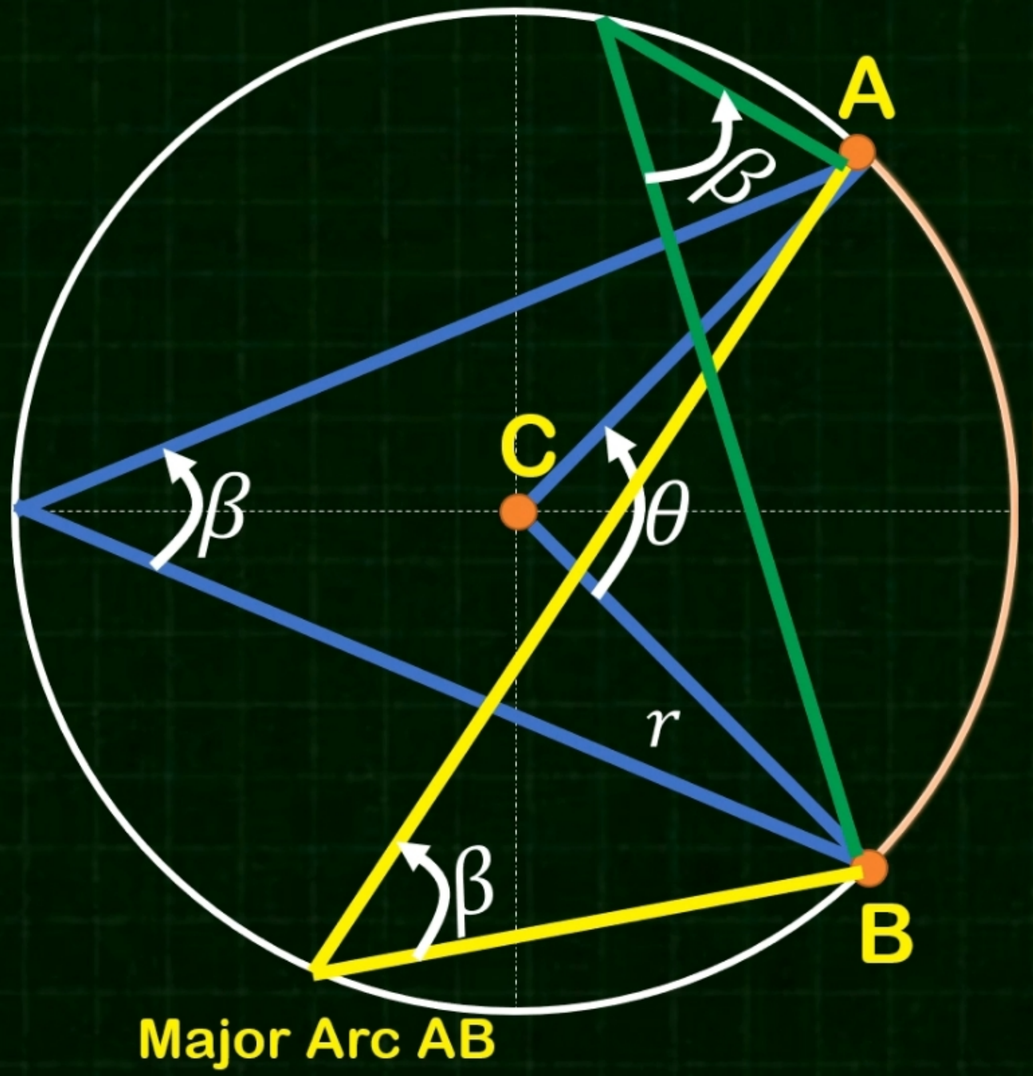
CIRCLE



Peripheral Angle Theorem

$$\theta = 2\beta \quad or \quad \beta = \theta/2$$

CIRCLE

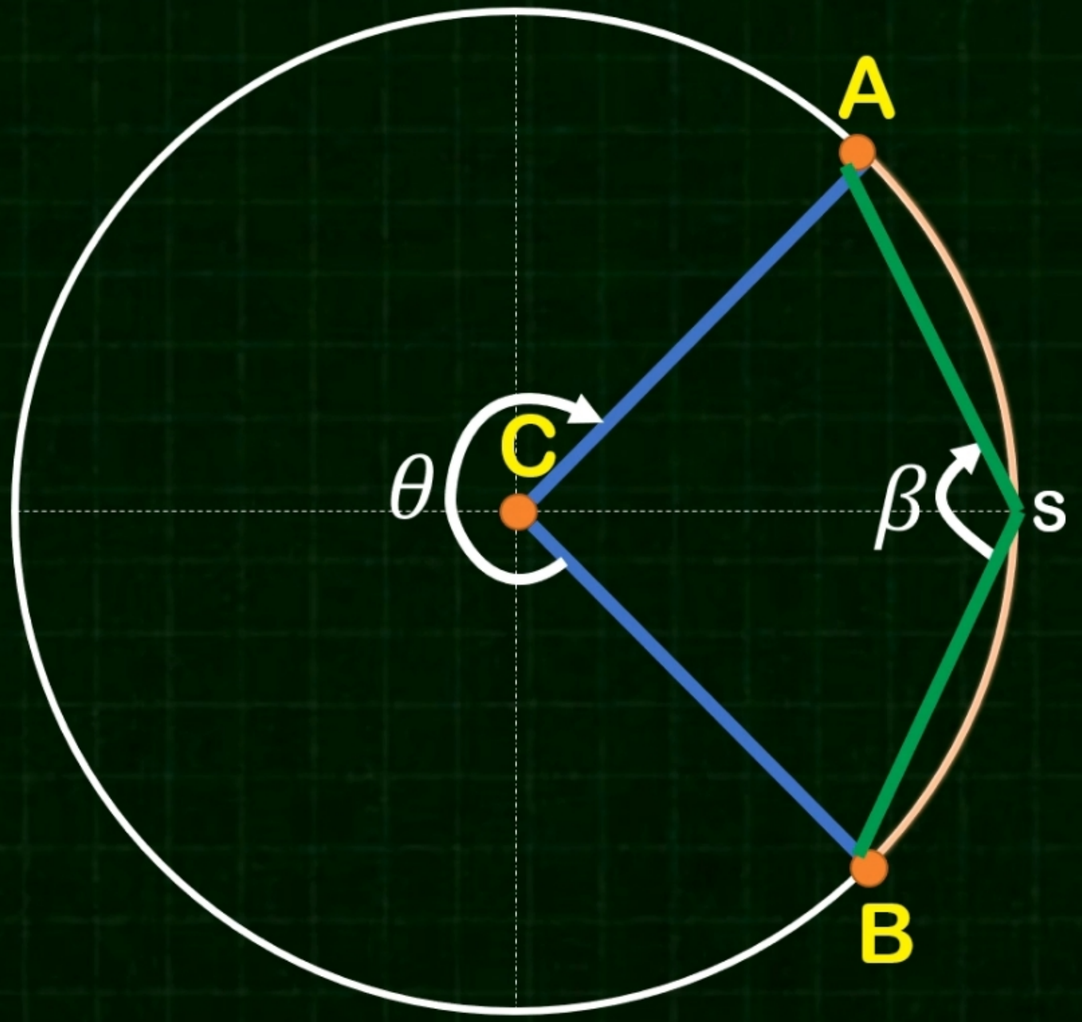


Arc Length $= S = \theta_{\text{radian}} R$

Peripheral Angle Theorem

$$\theta = 2\beta \quad or \quad \beta = \theta/2$$

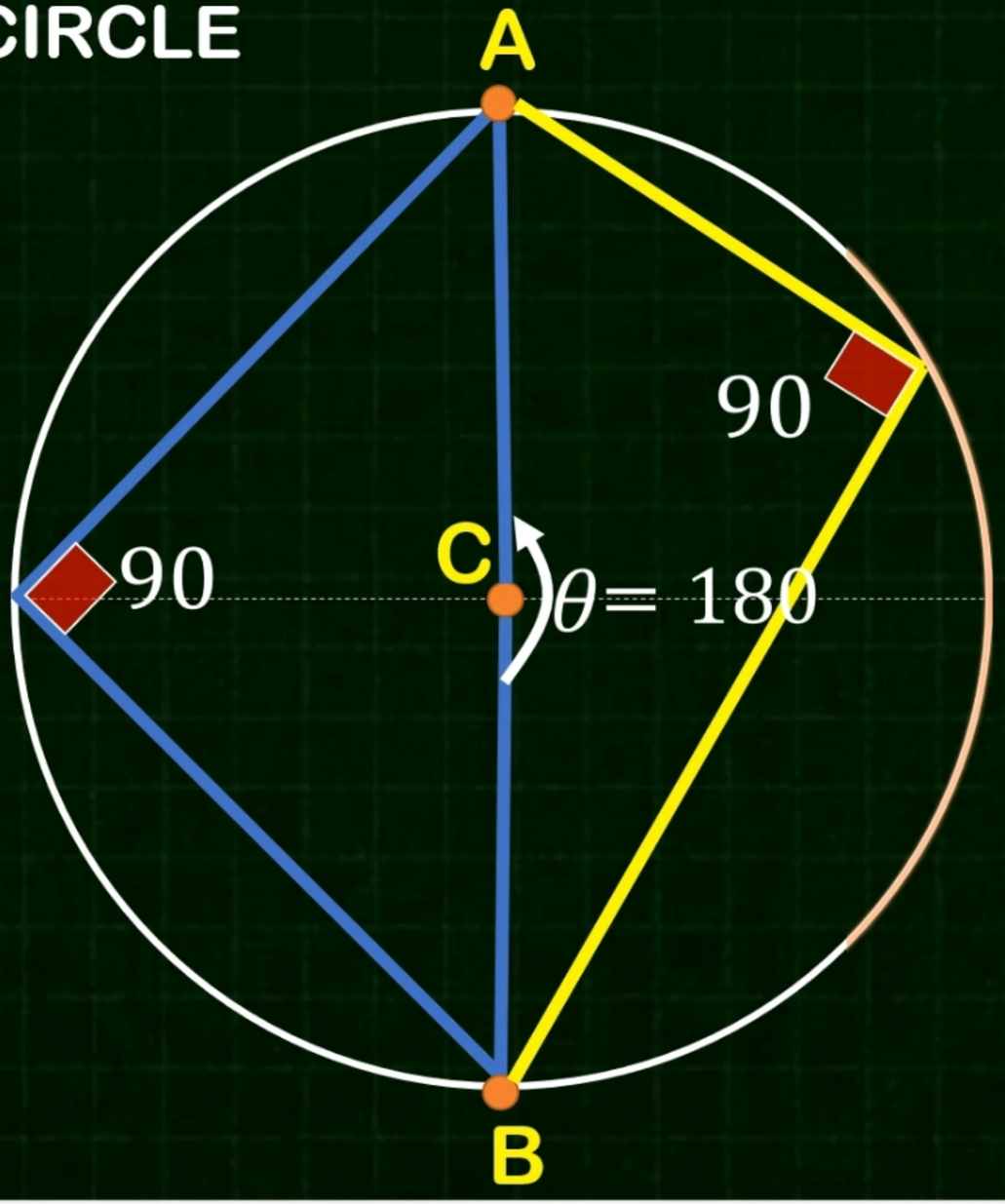
CIRCLE



Peripheral Angle Theorem

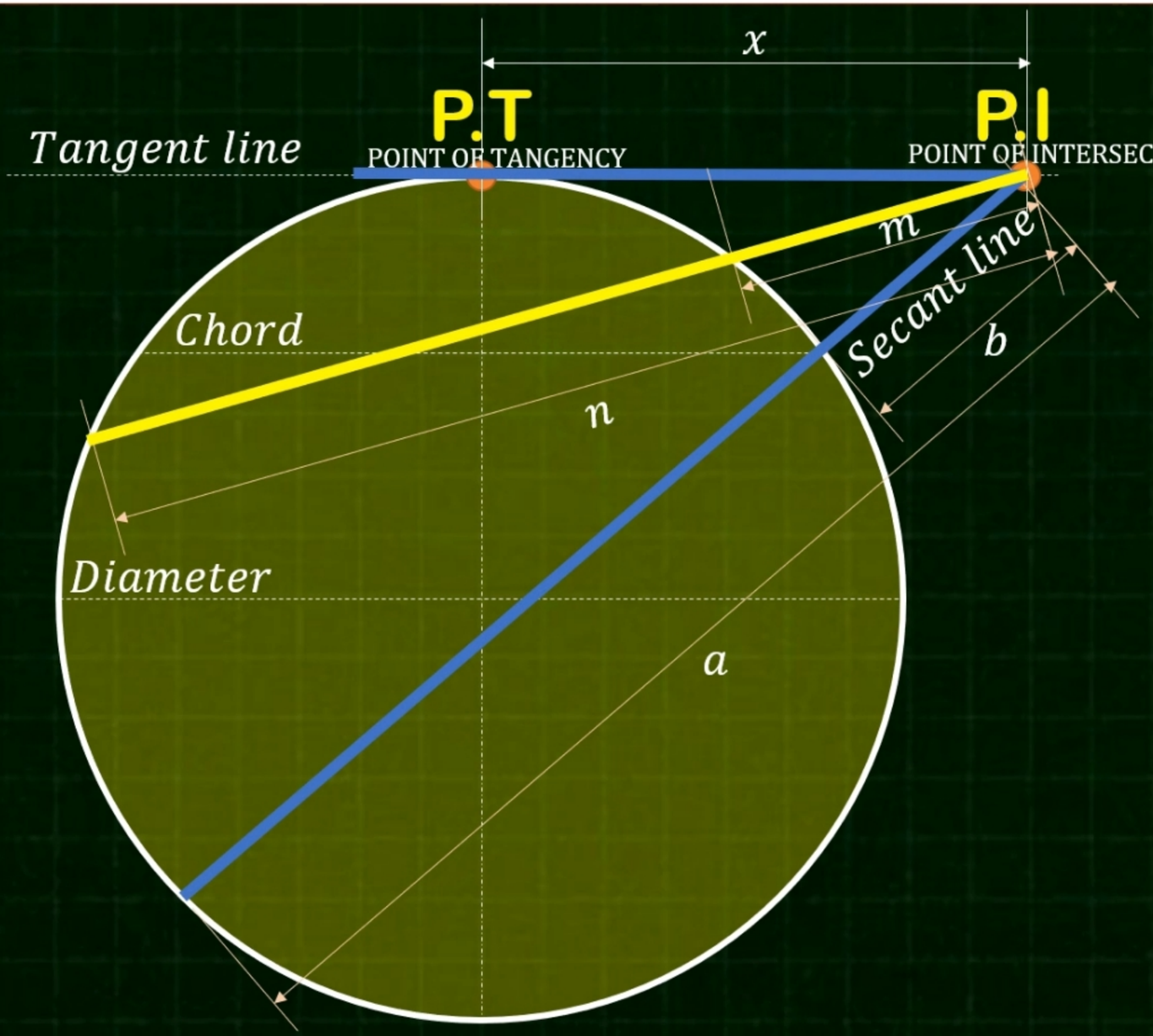
$$\theta = 2\beta \quad or \quad \beta = \theta/2$$

CIRCLE



Peripheral Angle Theorem

$$\theta = 2\beta \quad or \quad \beta = \theta/2$$

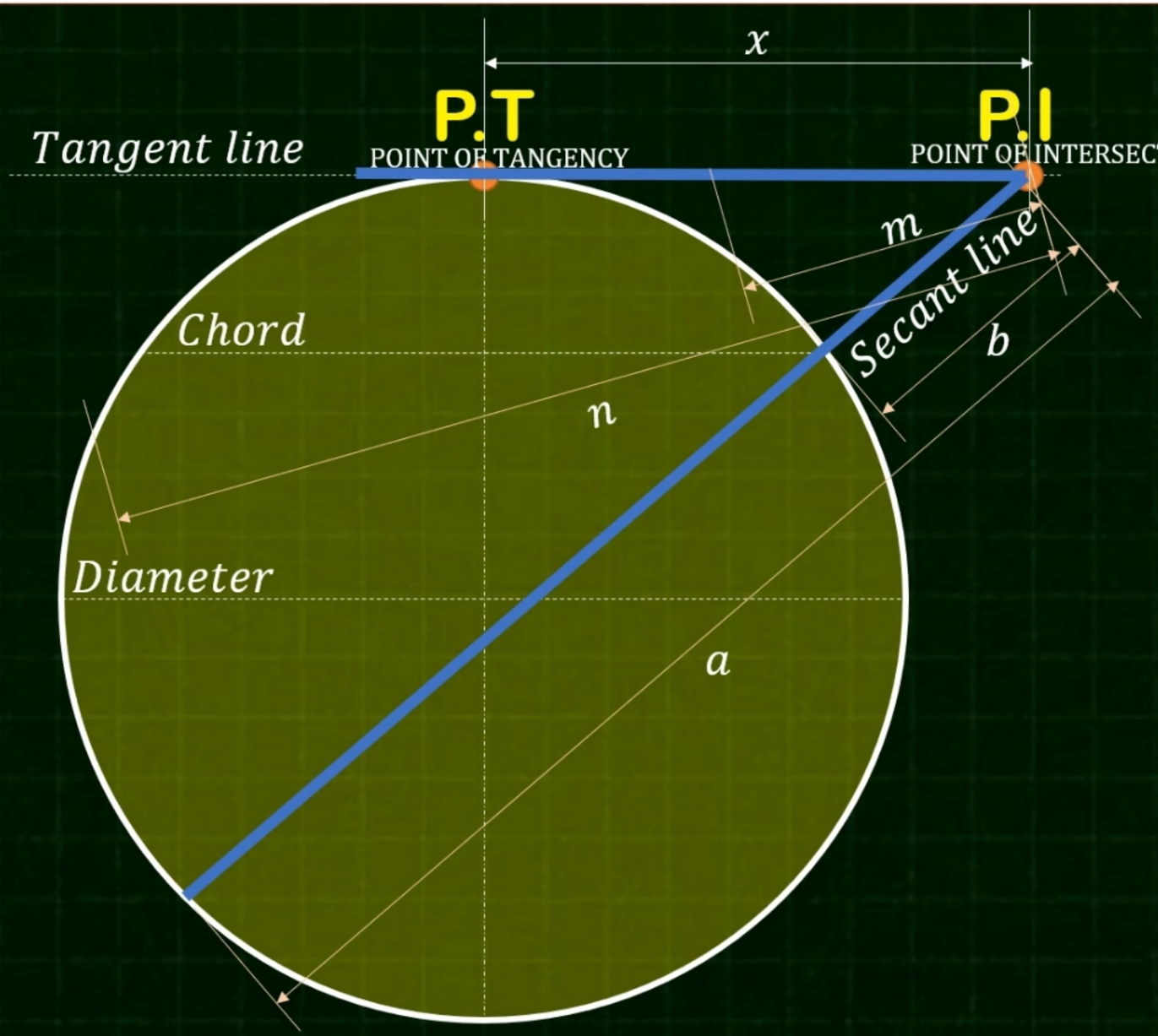


TANGENT – SECANT THEOREM

$$x^2 = ab$$

$$x^2 = mn$$

$$mn = ab$$

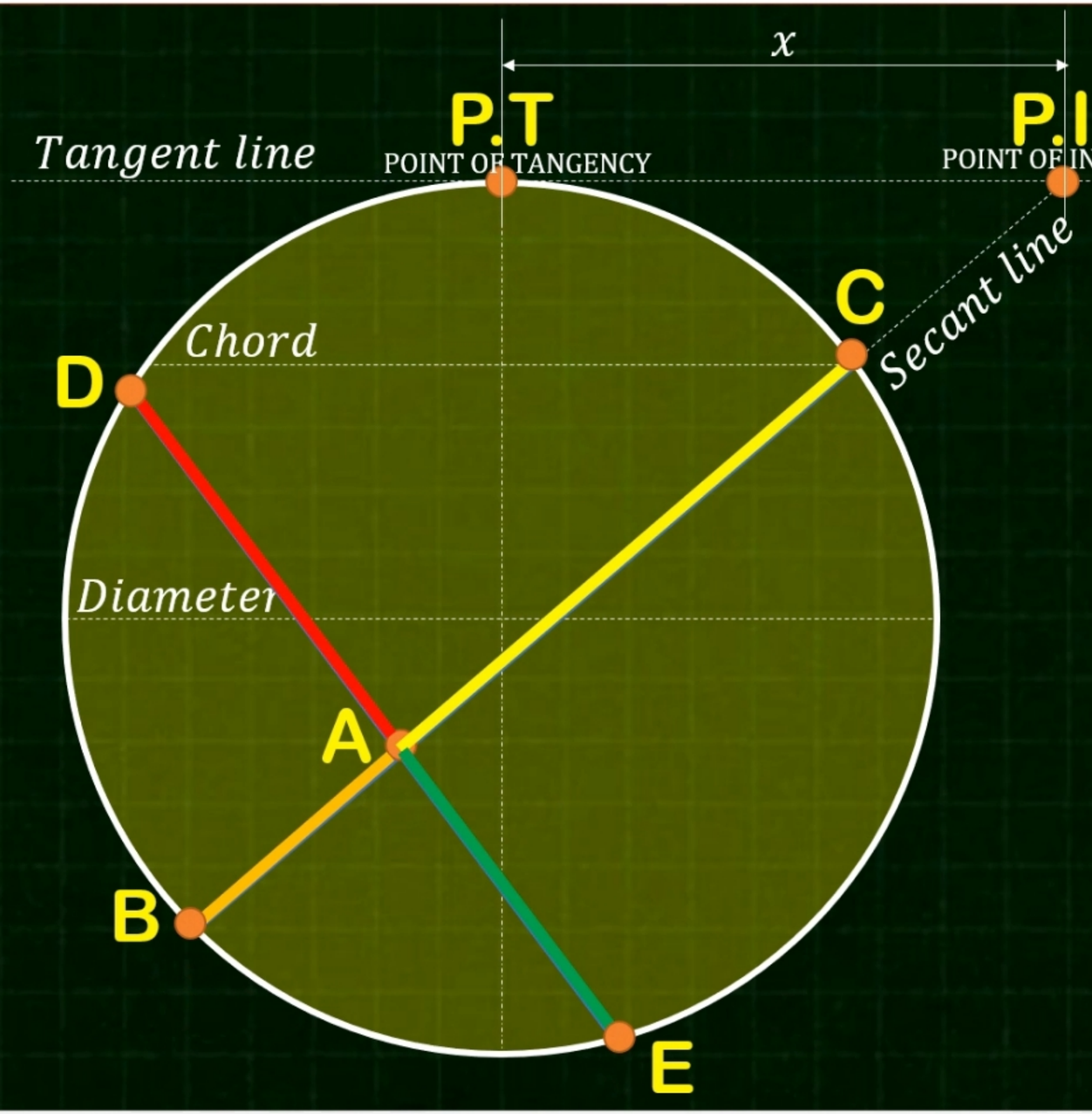


TANGENT – SECANT THEOREM

$$x^2 = ab$$

SECANT-SECANT THEOREM

$$mn = ab$$



TANGENT – SECANT THEOREM

$$x^2 = ab$$

SECANT-SECANT THEOREM

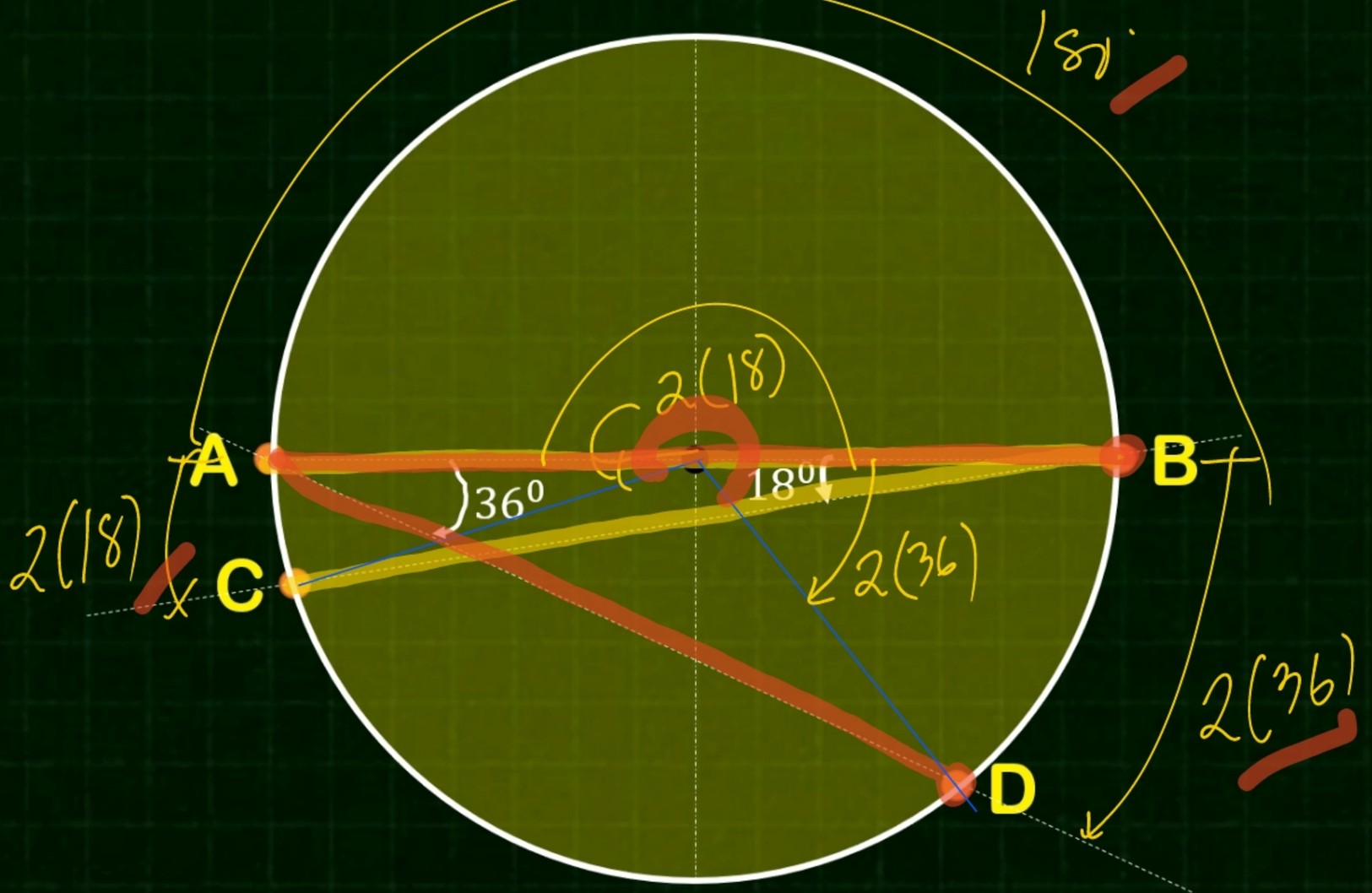
$$mn = ab$$

CROSS – CHORD THEOREM

$$(AB)(AC) = (AD)(AE)$$

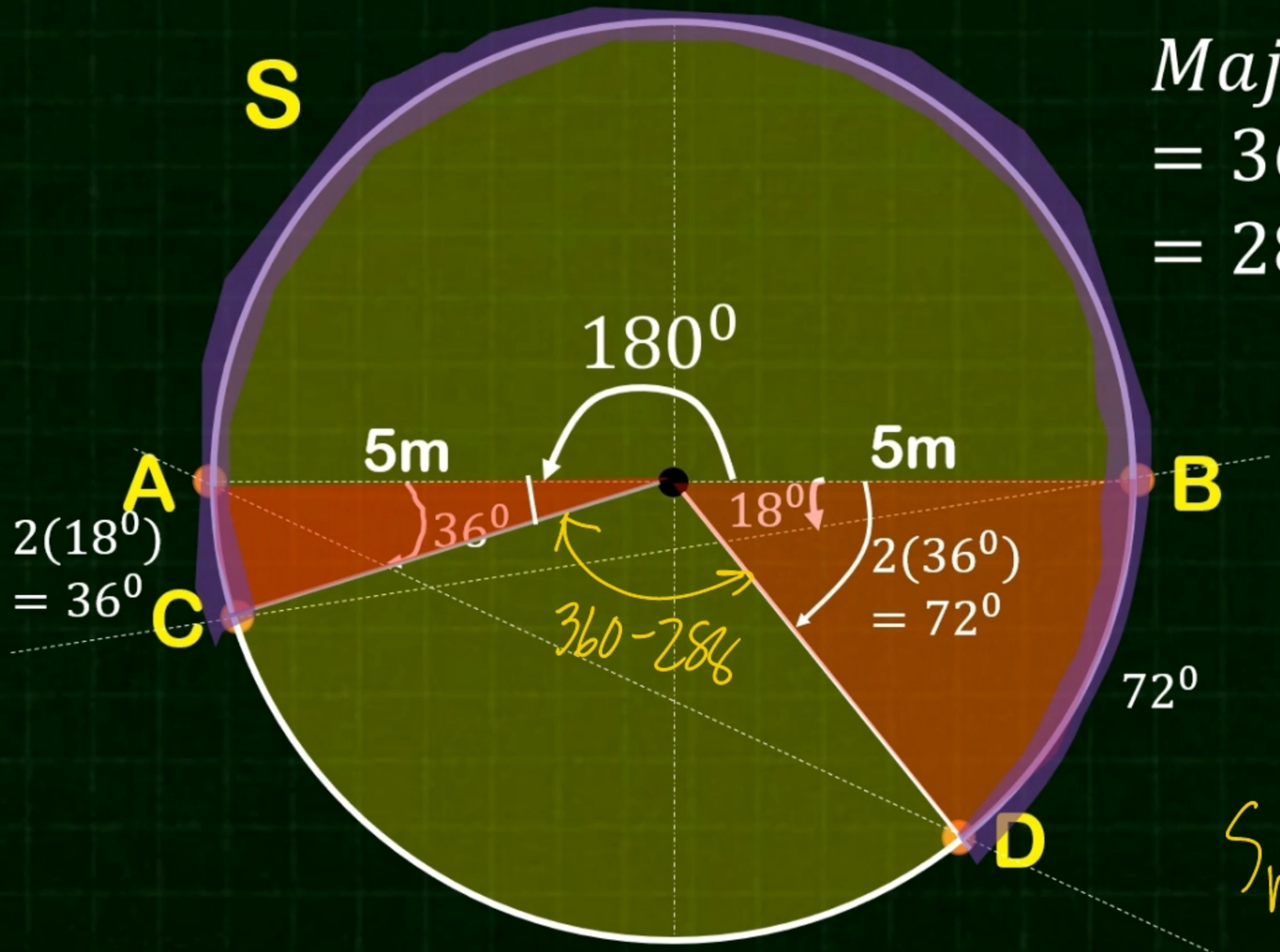
PAST BOARD EXAM:

SOLVE FOR THE LENGTH OF MAJOR ARC (CD). SEGMENT AB IS 10m



PAST BOARD EXAM:

SOLVE FOR THE LENGTH OF MAJOR ARC (CD).
SEGMENT AB IS 10m



SOLUTION:

By Peripheral Angle Theorem

$$Major\ Arc\ BC$$
$$= 36 + 180 + 72$$
$$= 288^0$$

$$S = r\theta^{\text{rad}}$$

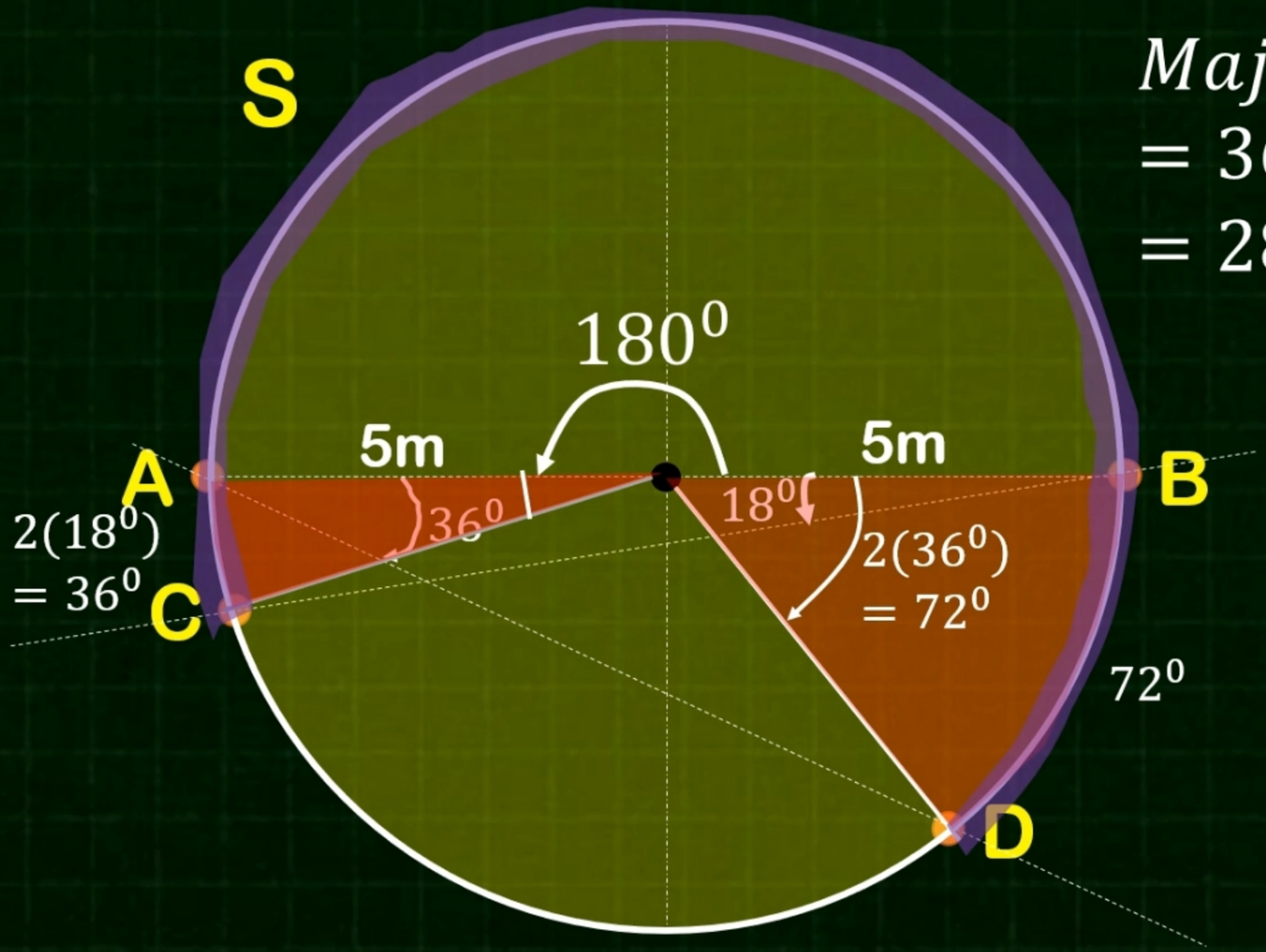
$$S = 5(288)\left(\frac{\pi}{180}\right)$$

$$\boxed{S = 25.13m}$$

$$S_{minor} = 5(360 - 288)\left(\frac{\pi}{180}\right)$$
$$=$$

PAST BOARD EXAM:

SOLVE FOR THE LENGTH OF MAJOR ARC (CD).
SEGMENT AB IS 10m



SOLUTION:

By Peripheral Angle Theorem

$$Major\ Arc\ BC$$
$$= 36 + 180 + 72$$
$$= 288^0$$

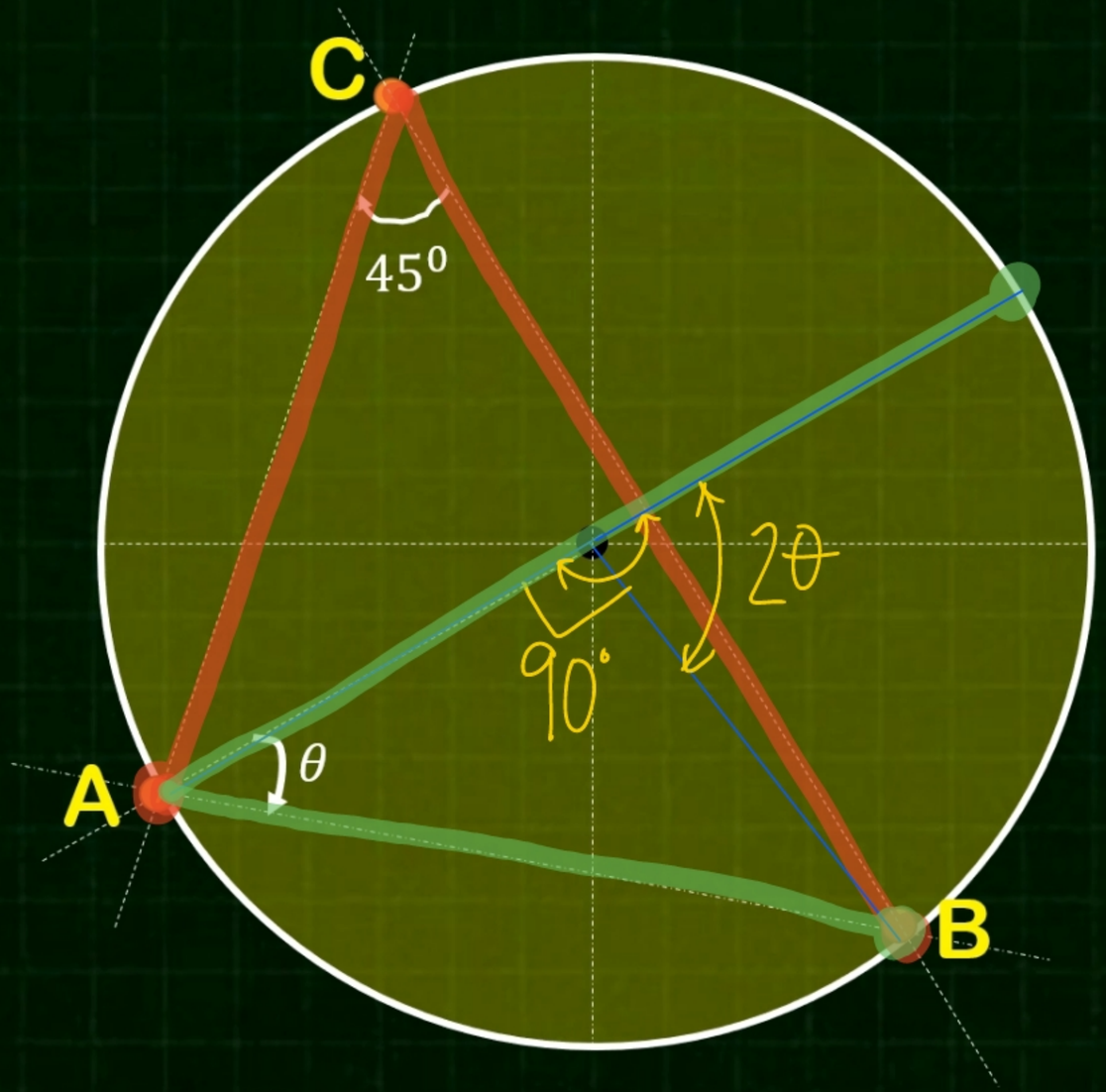
$$S = r\theta^{\text{rad}}$$

$$S = 5(288)\left(\frac{\pi}{180}\right)$$

$$\boxed{S = 25.13m}$$

PAST BOARD EXAM:

SOLVE FOR θ



$180 = 90 + 2\theta$

$\boxed{\theta = 45°}$

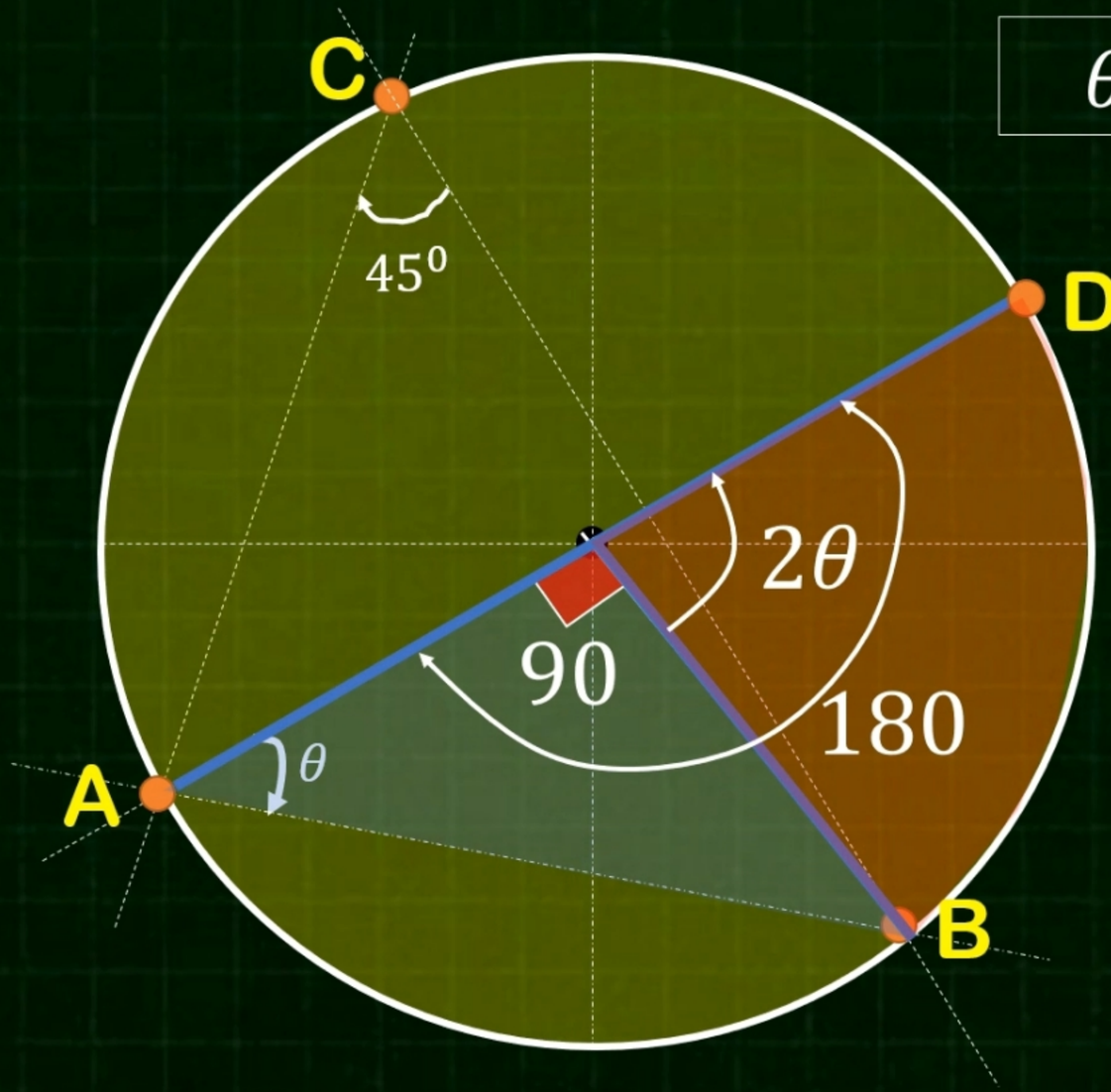
ans.

SOLUTION:

By Peripheral Angle Theorem

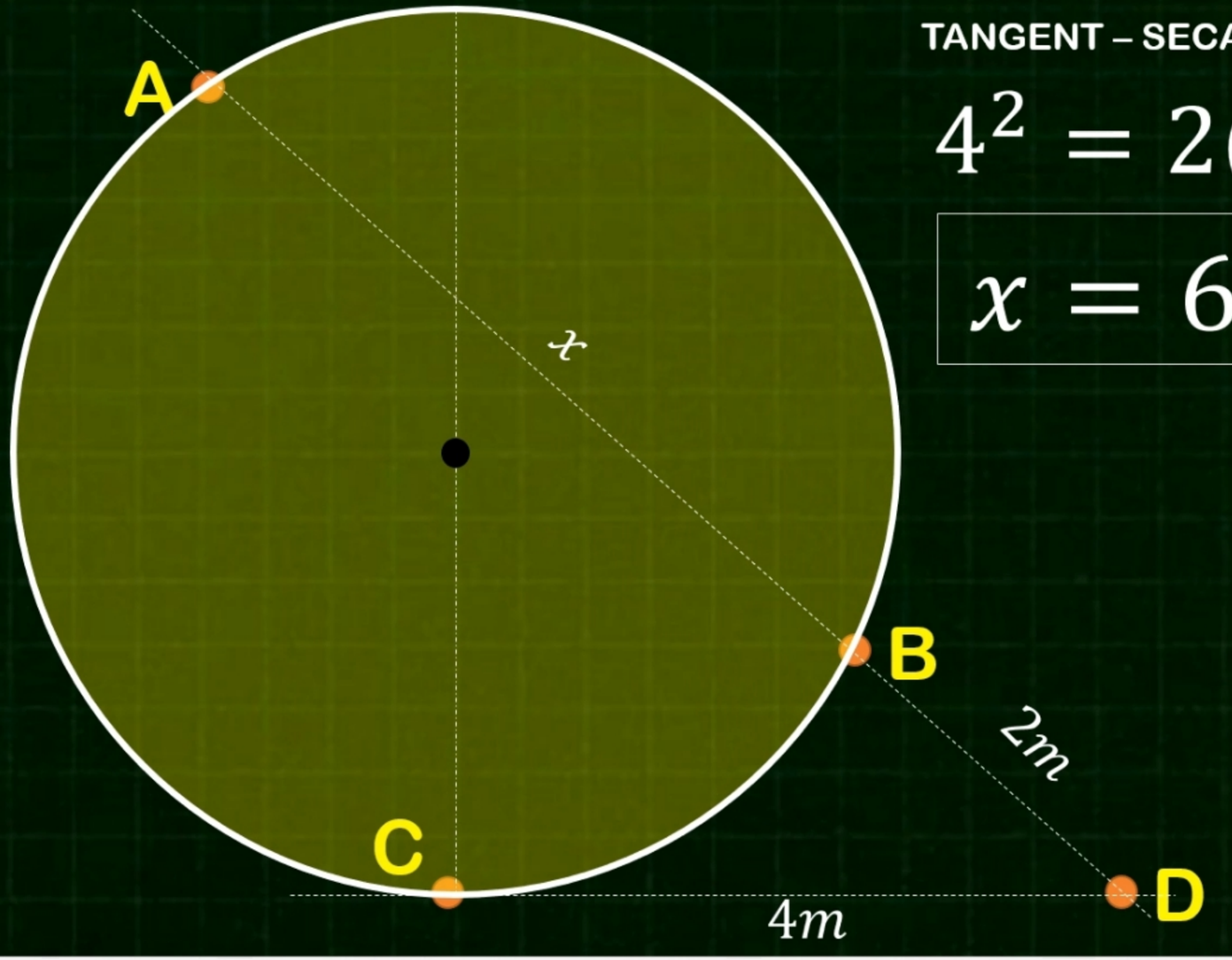
$$90 + 2\theta = 180$$

$$\theta = 45^0$$



PAST BOARD EXAM:

Solve for x:



SOLUTION:

TANGENT – SECANT THEOREM

$$4^2 = 2(2 + x)$$

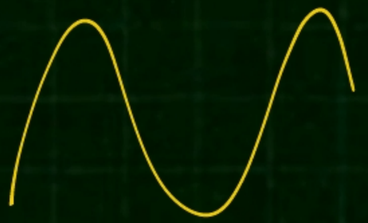
$$\boxed{x = 6m}$$

TRIGONOMETRIC EQUATIONS

TRIGONOMETRIC EQUATION

$$y = a \sin \left[\frac{2\pi}{b}(x + p)\right] + d$$

cos
tan
cot
sec
csc



TRIGONOMETRIC EQUATION

$$y = a \sin\left[\frac{2\pi}{b}(x + p)\right] + d$$

$y - d$

VERTICAL SHIFT

PHASE SHIFT

PERIOD

AMPLITUDE

TRIGONOMETRIC EQUATION

$$y = a \sin\left[\frac{2\pi}{b}(x+p)\right] + d$$

Simulate

Run/Pause

$a \sin\left(\left(\frac{2\pi}{b}\right)(x+p)\right)+d$

$d = 7.33$

−10 10

$p = 0$

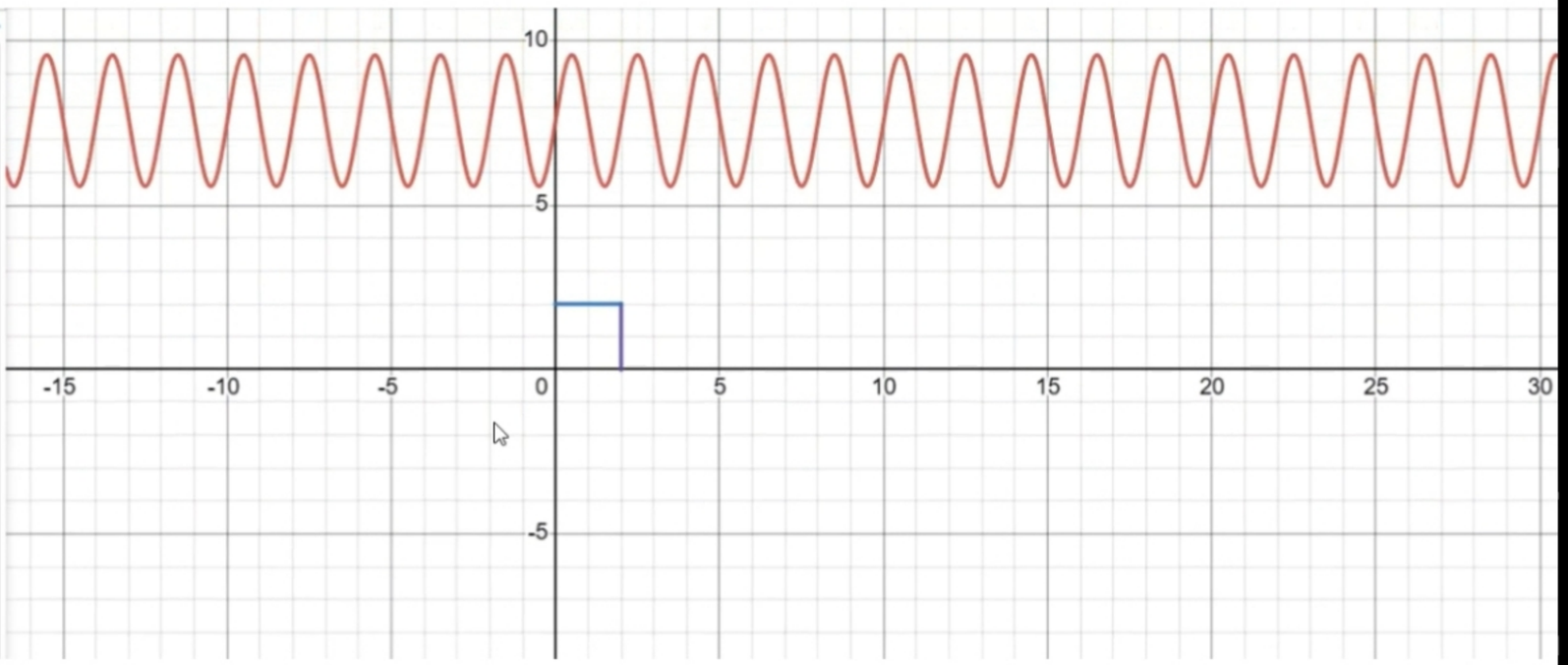
−10 10

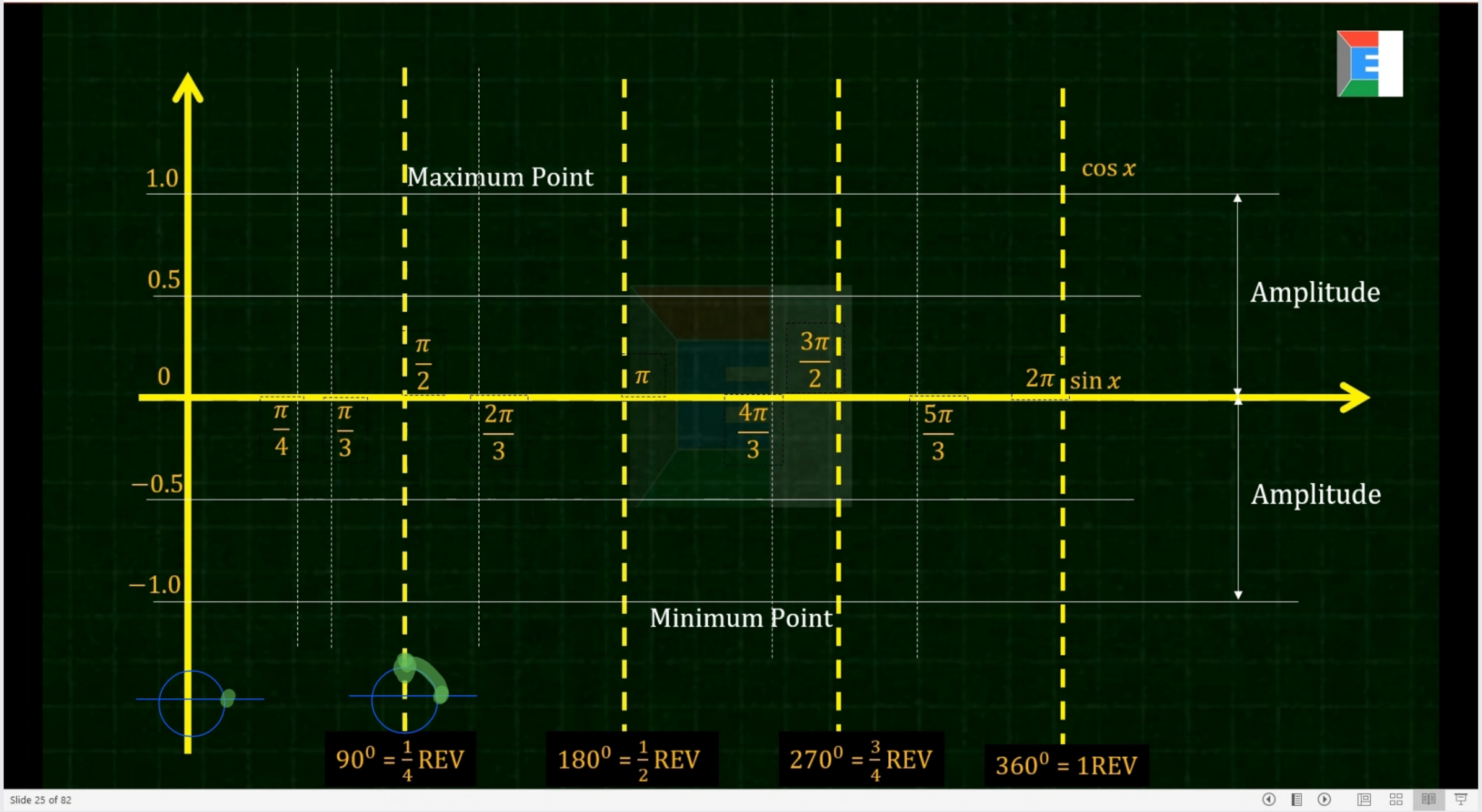
$a = 2$

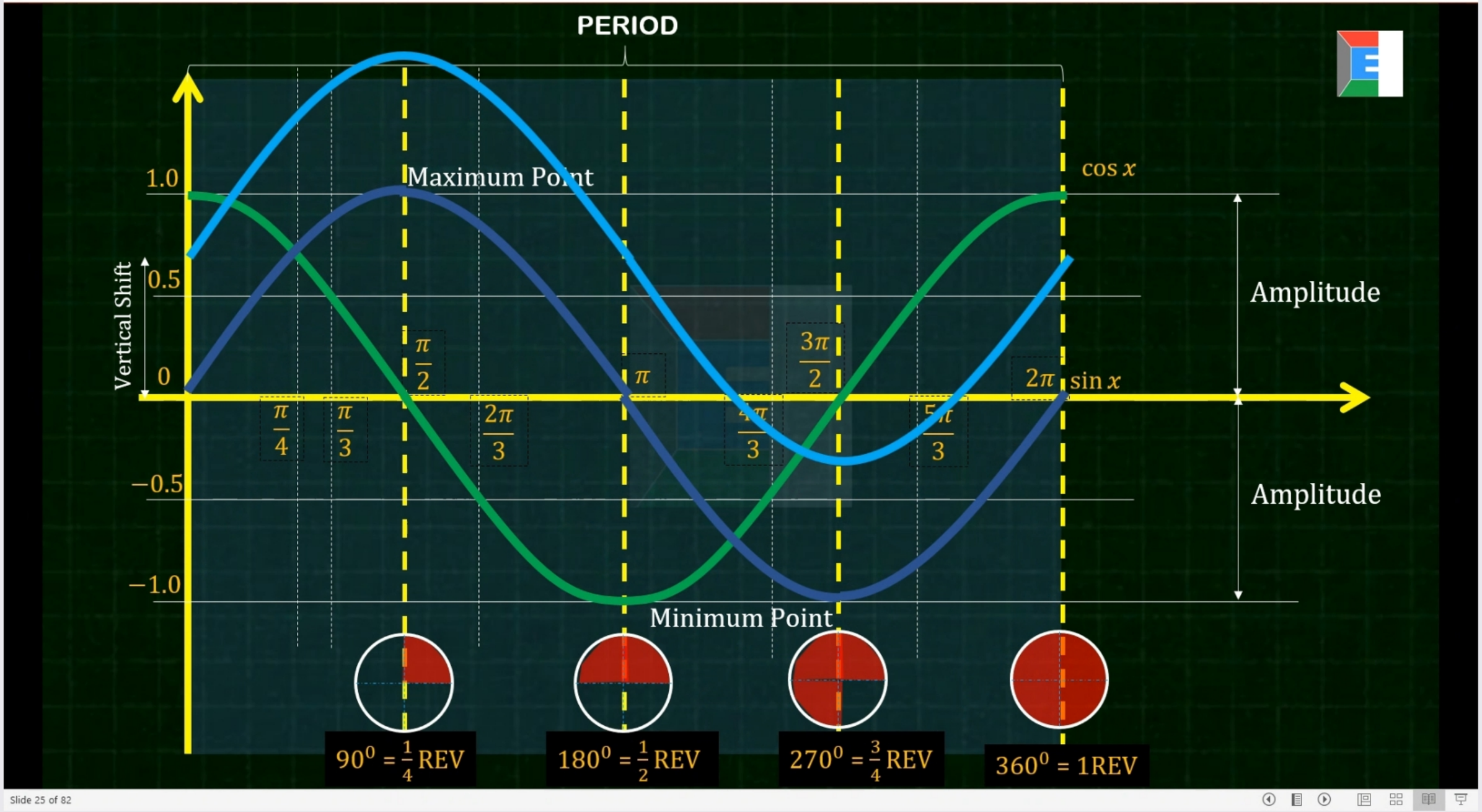
−10 10

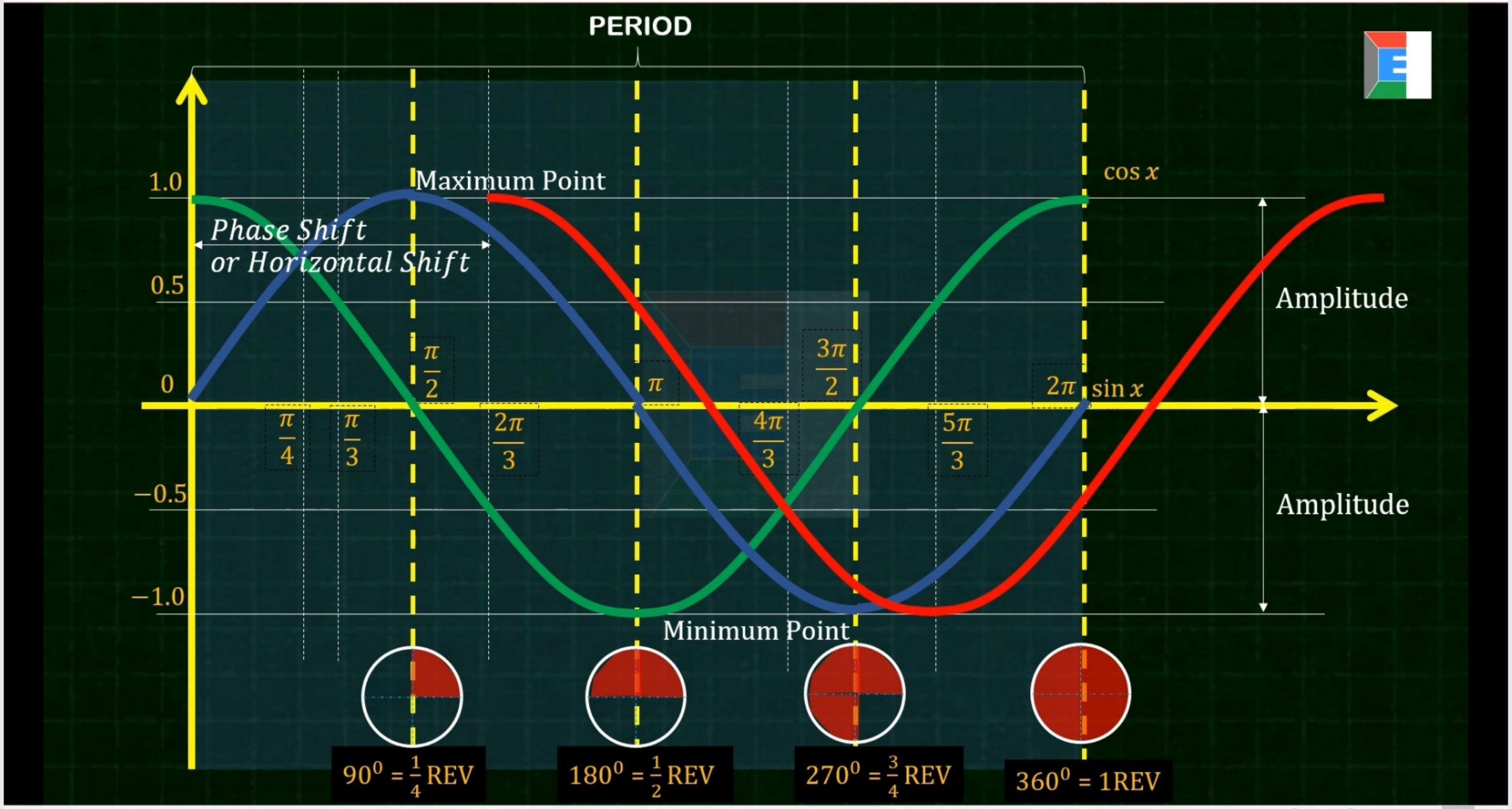
$b = 2$

−10 10

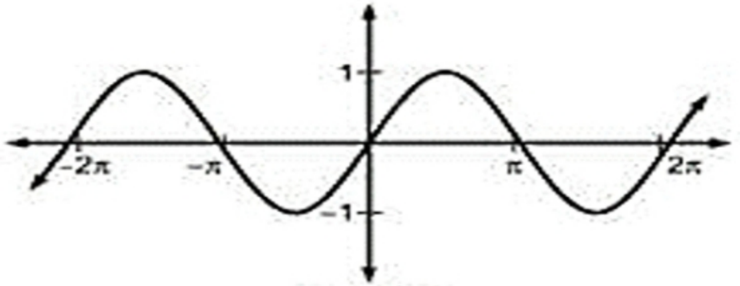



1.0
0.5
0
−0.5
−1.0
Maximum Point
Minimum Point
cos x
sin x
Amplitude
Amplitude
π/4
π/3
π/2
2π/3
π
4π/3
3π/2
5π/3
2π
90⁰ = 1/4 REV
180⁰ = 1/2 REV
270⁰ = 3/4 REV
360⁰ = 1REV

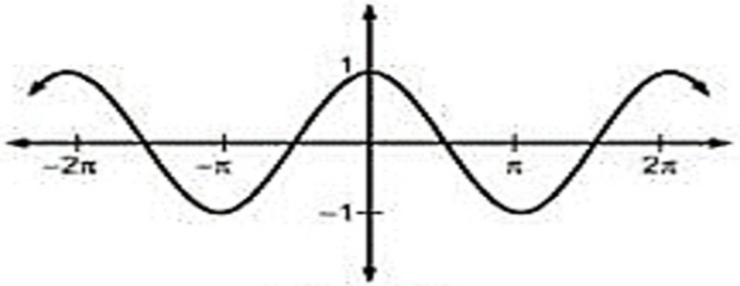

PERIOD
Maximum Point
Minimum Point
1.0
0.5
0
−0.5
−1.0
Vertical Shift
cos x
sin x
Amplitude
Amplitude
$\frac{\pi}{4}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
π
$\frac{4\pi}{3}$
$\frac{3\pi}{2}$
$\frac{5\pi}{3}$
2π
$90^0 = \frac{1}{4}$ REV
$180^0 = \frac{1}{2}$ REV
$270^0 = \frac{3}{4}$ REV
360^0 = 1REV


PERIOD
1.0
0.5
0
−0.5
−1.0
Maximum Point
Minimum Point
Phase Shift
or Horizontal Shift
cos x
sin x
Amplitude
Amplitude
π/4
π/3
π/2
2π/3
π
4π/3
3π/2
5π/3
2π
90⁰ = 1/4 REV
180⁰ = 1/2 REV
270⁰ = 3/4 REV
360⁰ = 1REV

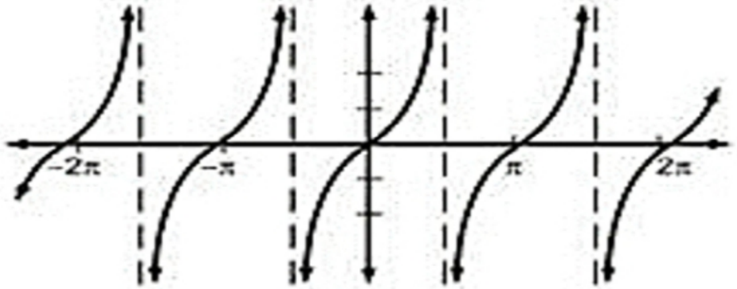
TRIGONOMETRIC CURVES



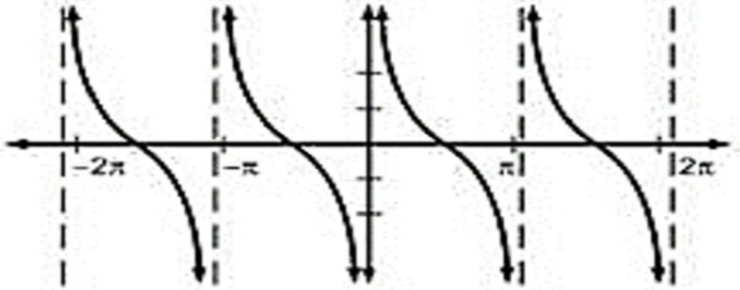
$y = \sin x$



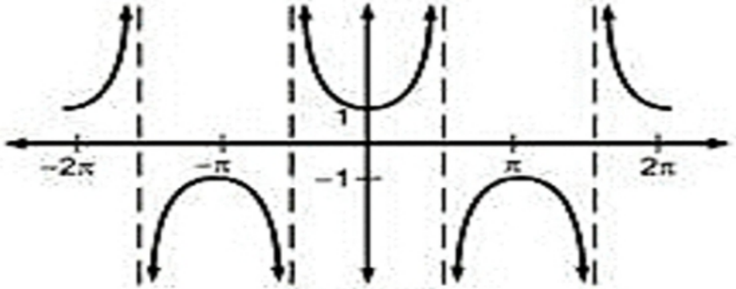
$y = \cos x$



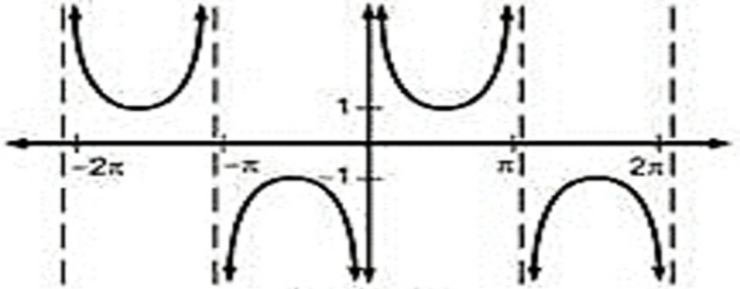
$y = \tan x$



$y = \cot x$

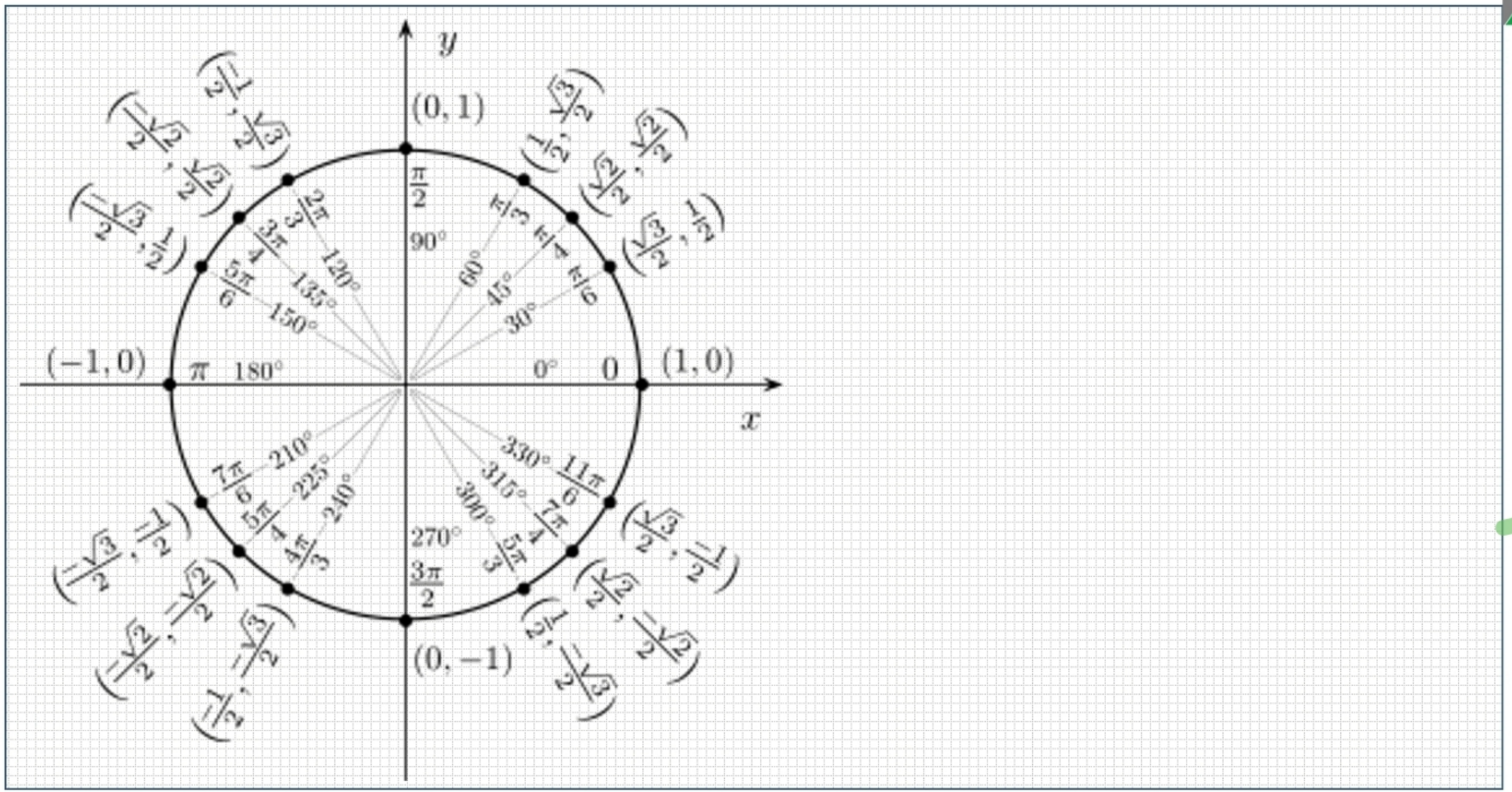


$y = \sec x$

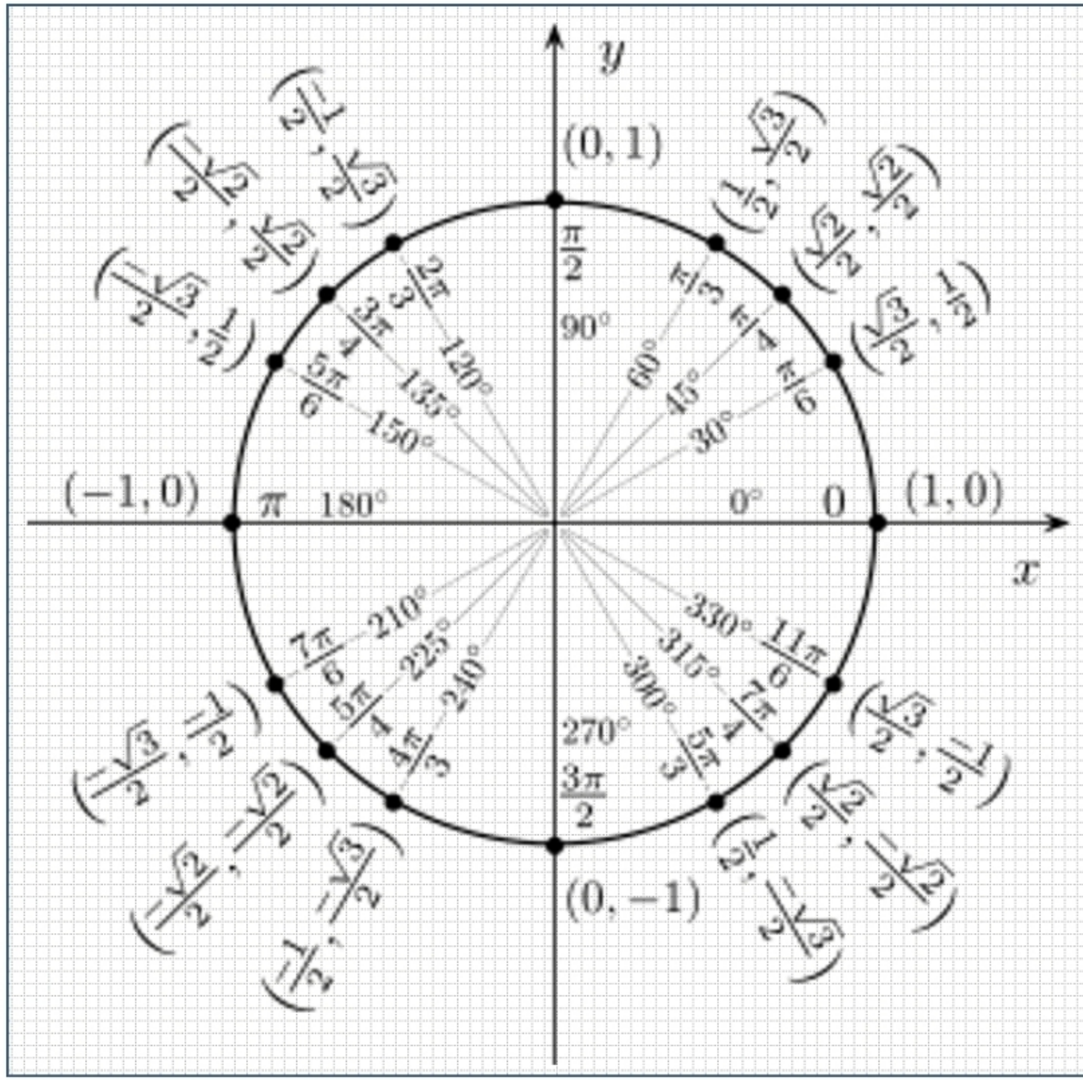


$y = \csc x$

TRIGONOMETRIC FUNCTIONS



TRIGONOMETRIC FUNCTIONS



Sini

Quadrant II		Quadrant I	
SUGAR		ADD	
sin, cosec	+	sin, cosec	+
cos, sec	−	cos, sec	+
tan, cot	−	tan, cot	+
Quadrant III		**Quadrant IV**	
TO		COFFEE	
sin, cosec	−	sin, cosec	−
cos, sec	−	cos, sec	+
tan, cot	+	tan, cot	−

All

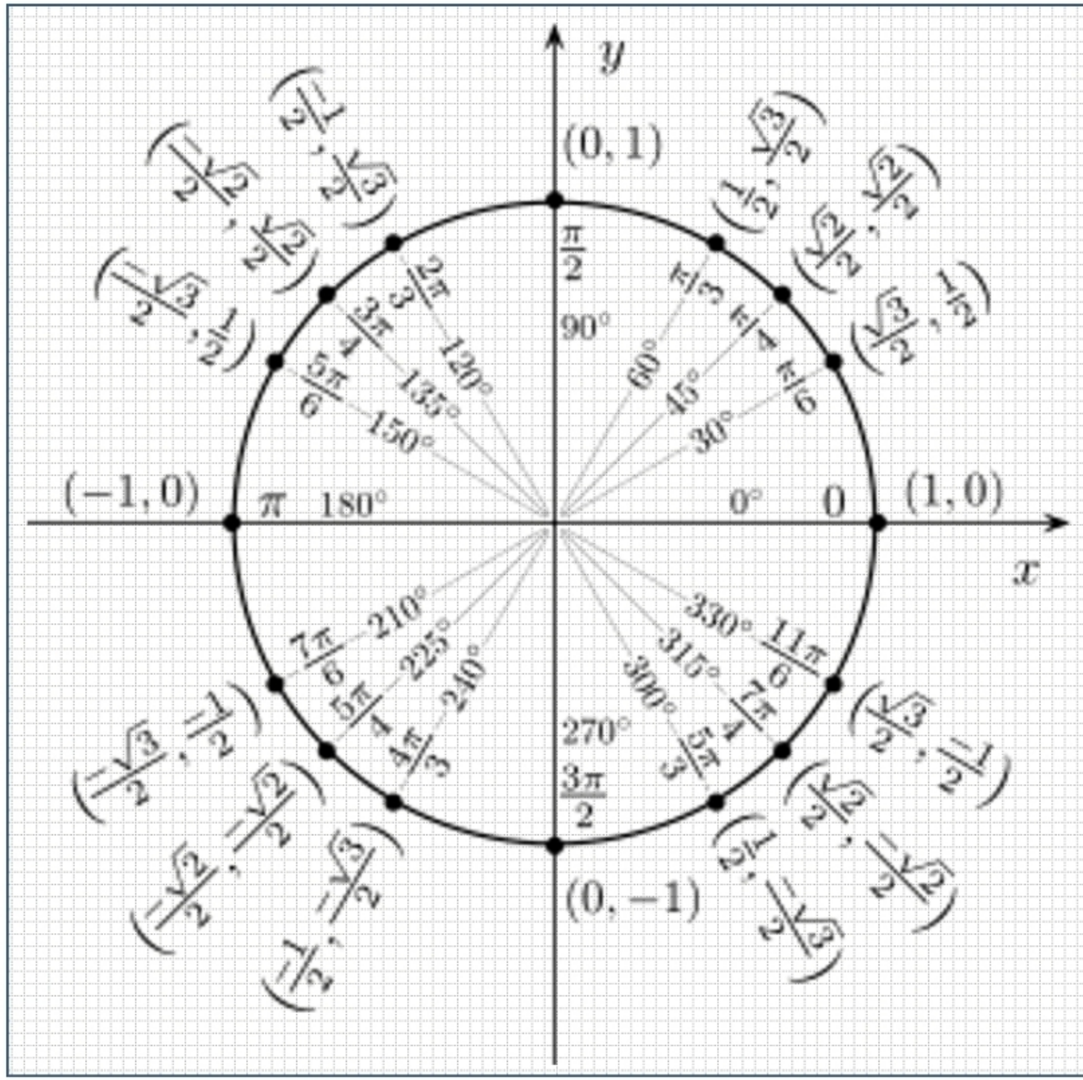
Cosini

Tan

TRIGONOMETRIC FUNCTIONS

Addt'l trigonometric functions:

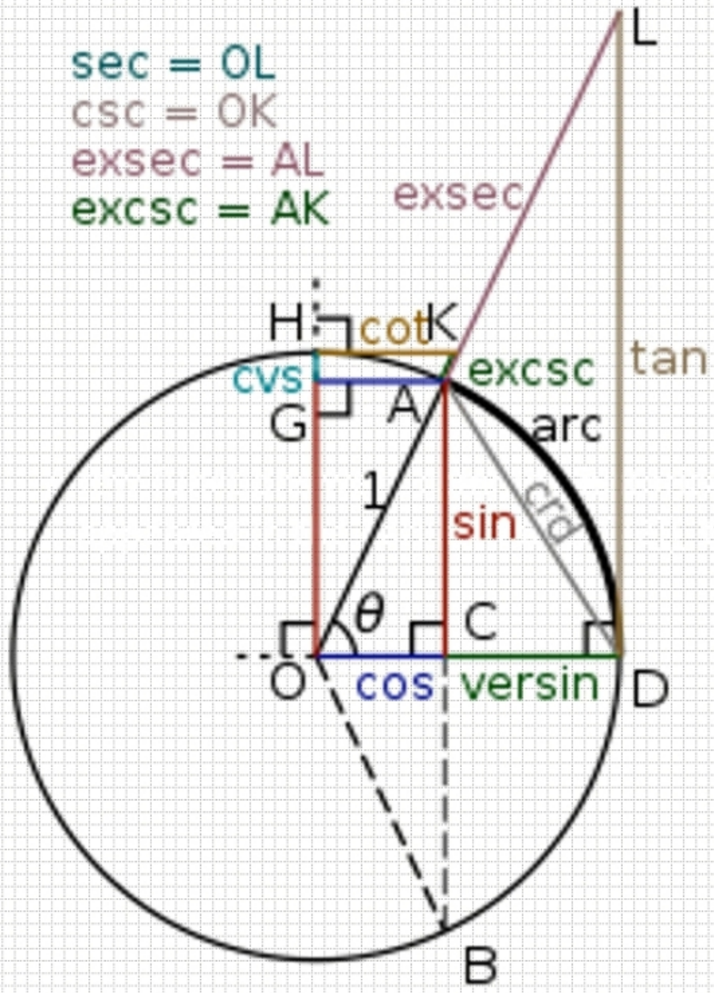
Archaic functions:



Quadrant II	Quadrant I
SUGAR	ADD
sin, cosec +	sin, cosec +
cos, sec −	cos, sec +
tan, cot −	tan, cot +

Quadrant III	Quadrant IV
TO	COFFEE
sin, cosec −	sin, cosec −
cos, sec −	cos, sec +
tan, cot +	tan, cot −

OTHER TRIGONOMETRIC FUNCTIONS



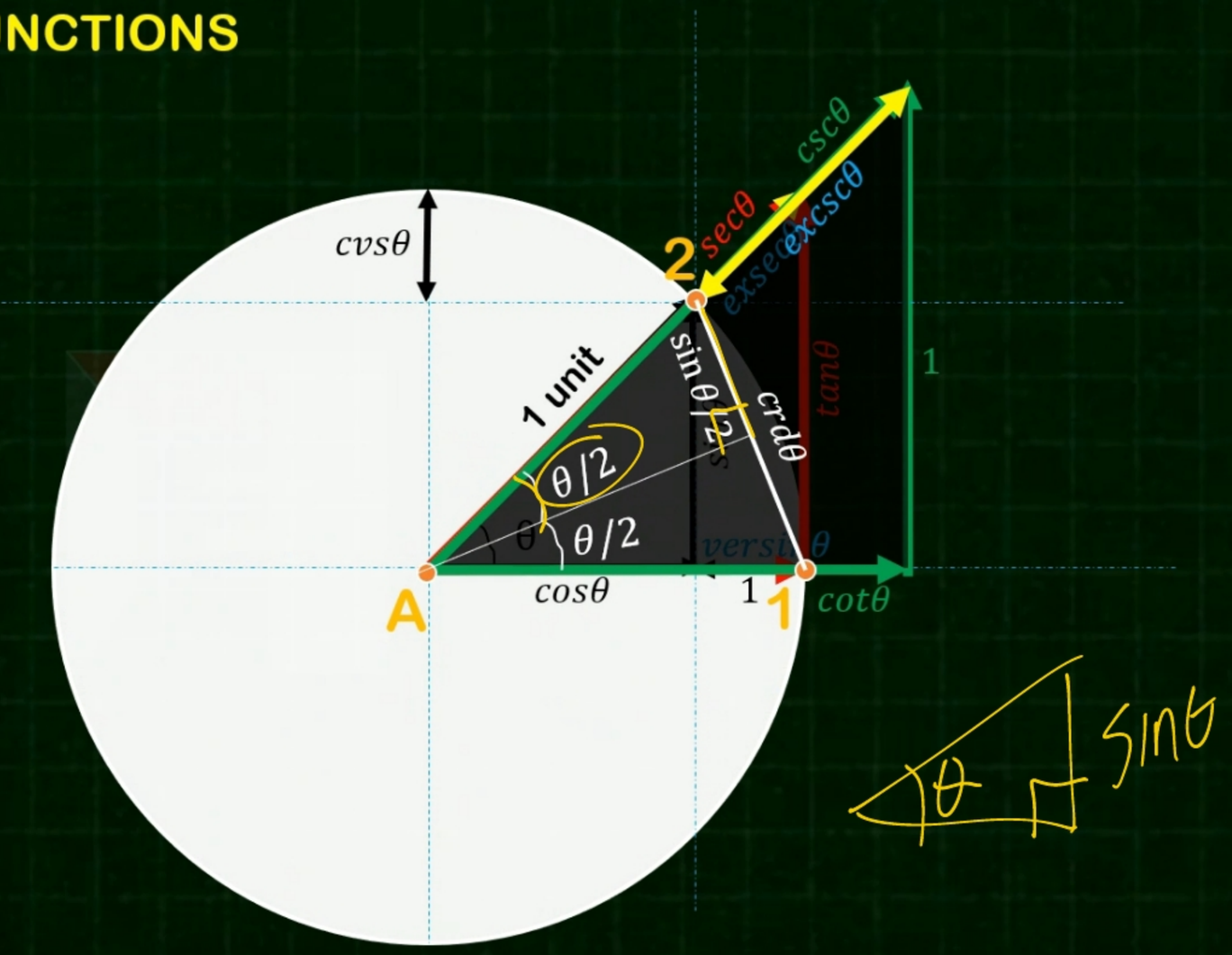
A few functions were common historically, but are now seldom used, such as the **chord**, the **versine**, the **coversine**, the **haversine**,the **exsecant** and the **excosecant**.

- $\text{crd}(\theta) = 2\sin(\frac{\theta}{2})$
- $\text{versin}(\theta) = 1 - \cos(\theta) = 2\sin^2(\frac{\theta}{2})$
- $\text{coversin}(\theta) = 1 - \sin(\theta) = \text{versin}(\frac{\pi}{2} - \theta)$
- $\text{haversin}(\theta) = \frac{1}{2}\text{versin}(\theta) = \sin^2(\frac{\theta}{2})$
- $\text{exsec}(\theta) = \sec(\theta) - 1$
- $\text{excsc}(\theta) = \text{exsec}(\frac{\pi}{2} - \theta) = \csc(\theta) - 1$

OTHER TRIGONOMETRIC FUNCTIONS

$$coversin\theta = 1 - sin\theta$$
$$versin\theta = 1 - cos\theta$$

$$exsec\theta = sec\theta - 1$$
$$excsc\theta = csc\theta - 1$$



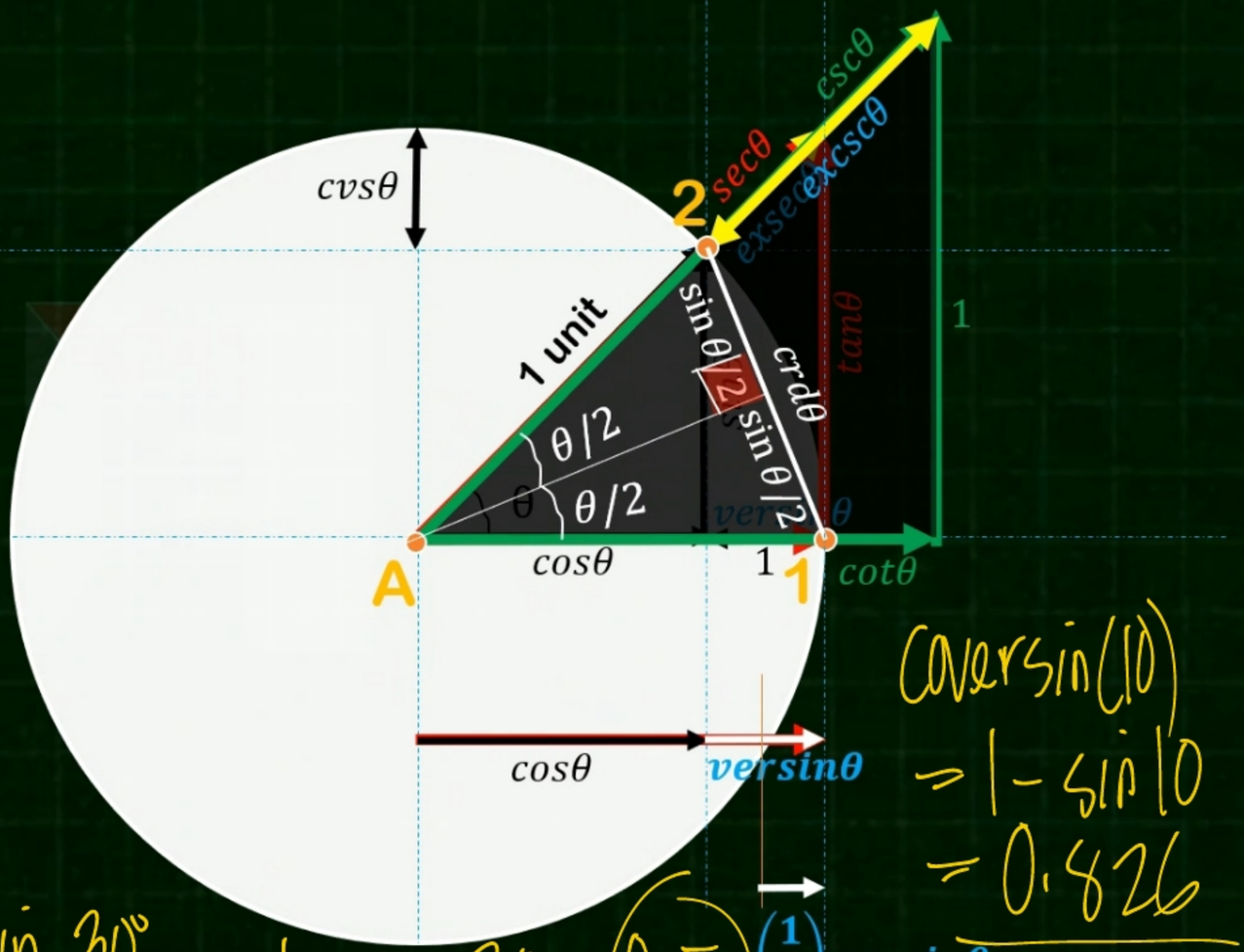
OTHER TRIGONOMETRIC FUNCTIONS

$$coversin\theta = 1 - sin\theta$$
$$versin\theta = 1 - cos\theta$$

$$exsec\theta = sec\theta - 1$$
$$excsc\theta = csc\theta - 1$$

$$crd\ \theta = 2\sin\left(\frac{\theta}{2}\right)$$

$$haversin\ \theta = \left(\frac{1}{2}\right) versin\ \theta$$



Coversin 30° = 1 − sin 30 = 0.5

Coversin(1.0)
= 1 − sin 1.0
= 0.826

PAST CE BOARD EXAM:

A Ferris wheel has a diameter of 30m with a center 17m above the ground. It makes one complete rotation every 60seconds. Assumed a rider starts at the lowest point. Which of the following equation best describe the height of the rider with respect to time.

a) $h(t) = -17\cos\left(\frac{\pi t}{30}\right) + 15$

b) $h(t) = -30\cos\left(\frac{\pi t}{60}\right) + 17$

c) $h(t) = -15\cos\left(\frac{\pi t}{30}\right) + 17$

d) $h(t) = -30\cos\left(\frac{\pi t}{60}\right) + 17$

PAST CE BOARD EXAM:

A Ferris wheel has a diameter of 30m with a center 17m above the ground. It makes one complete rotation every 60seconds. Assumed a rider starts at the lowest point. Which of the following equation best describe the height of the rider from the ground with respect to time.

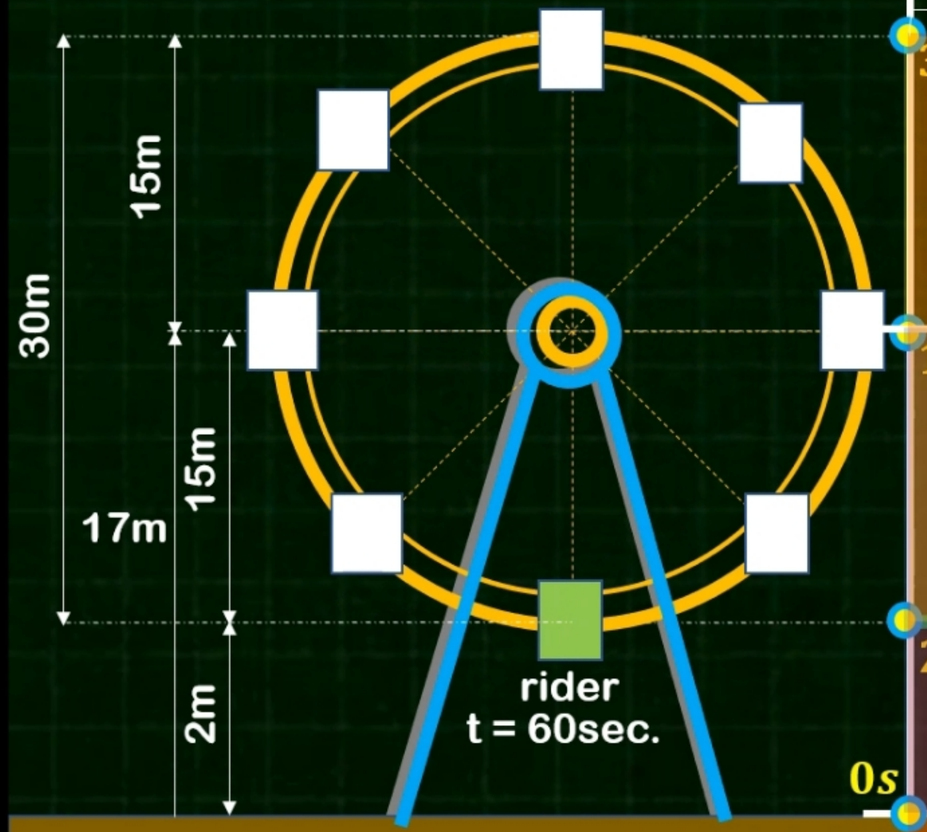
$$y = a\cos\left[\frac{2\pi}{b}(x+p)\right] + d$$

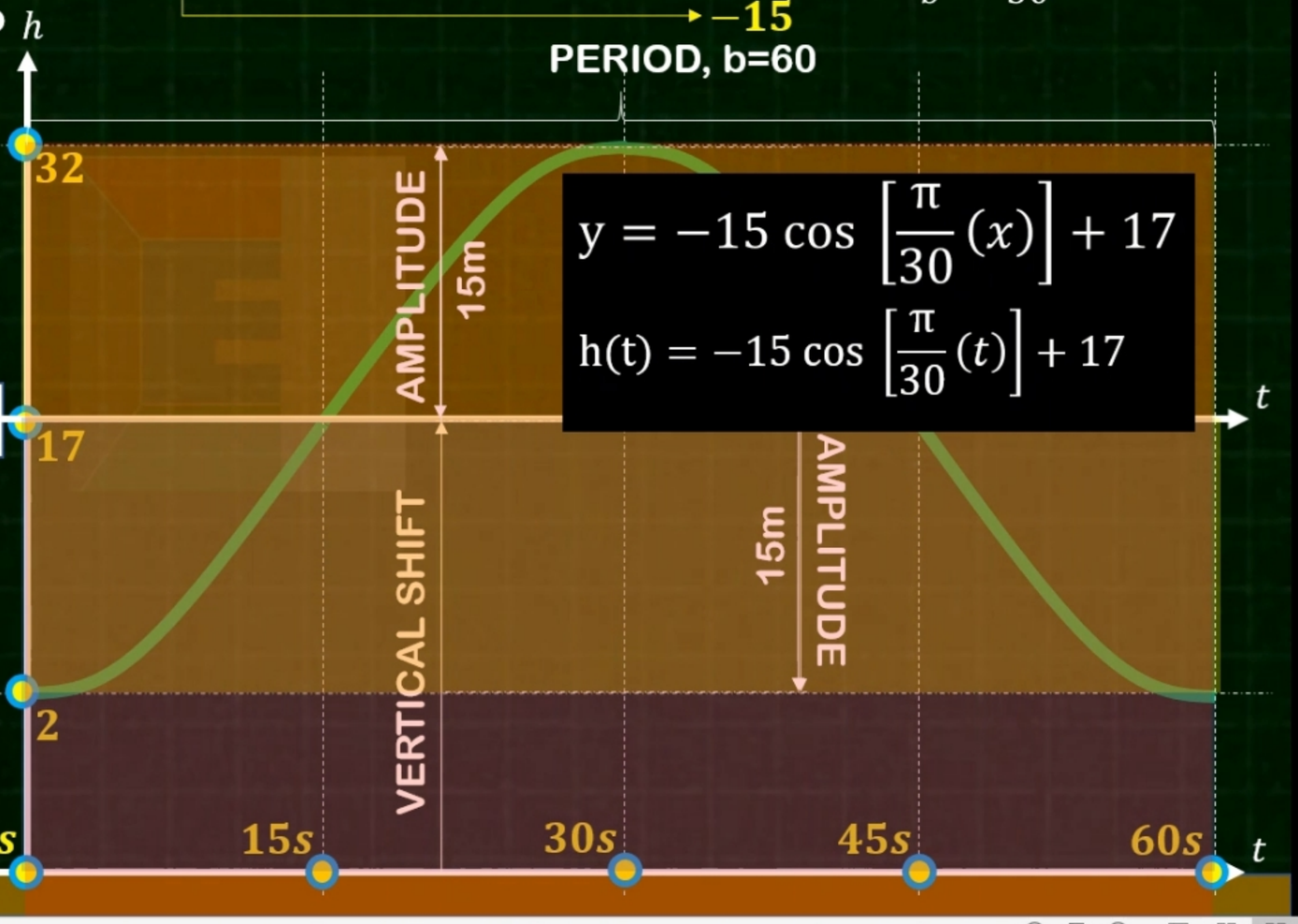
- d → 17
- p → 0
- a → −15

$$\frac{2\pi}{b} = \frac{2\pi}{60}$$

$$\frac{2\pi}{b} = \frac{\pi}{30}$$

PERIOD, b=60





$$y = -15\cos\left[\frac{\pi}{30}(x)\right] + 17$$

$$h(t) = -15\cos\left[\frac{\pi}{30}(t)\right] + 17$$

PAST CE BOARD EXAM:

A Ferris wheel has a diameter of 30m with a center 17m above the ground. It makes one complete rotation every 60seconds. Assumed a rider starts at the lowest point. Which of the following equation best describe the height of the rider with respect to time.

a) $h(t) = -17\cos\left(\frac{\pi t}{30}\right) + 15$

b) $h(t) = -30\cos\left(\frac{\pi t}{60}\right) + 17$

c) $h(t) = -15\cos\left(\frac{\pi t}{30}\right) + 17$

d) $h(t) = -30\cos\left(\frac{\pi t}{60}\right) + 17$

PAST CE BOARD EXAM:

A Ferris wheel has a diameter of 30m with a center 17m above the ground. It makes one complete rotation every 60seconds. Assumed a rider starts at the lowest point. What is the height of the rider at 45seconds?

a) $h(t) = 15m$

b) $h(t) = 17m$

c) $h(t) = 2m$

d) $h(t) = 32m$

SOLUTION:

$$h(t) = -15\cos\left(\frac{\pi t}{30}\right) + 17$$

$$h(45) = -15\cos\left(\frac{\pi(45)}{30}\right) + 17$$

$$h(45) = -15\cos\left(\frac{\pi(45)}{30}\, x \left(\frac{\mathbf{180^0}}{\boldsymbol{\pi}}\right)\right) + 17$$

$$\boldsymbol{h = 17m}$$

PAST CE BOARD EXAM:

A Ferris wheel has a diameter of 30m with a center 17m above the ground. It makes one complete rotation every 60seconds. Assumed a rider starts at the lowest point. Which of the following equation best describe the height of the rider from the ground with respect to time.

PAST CE BOARD EXAM:

A Ferris wheel has a diameter of 30m with a center 17m above the ground. It makes one complete rotation every 60seconds. Assumed a rider starts at the lowest point. At what time is the rider at a height of 15m?

a) $10.5s$

b) 11.7s

c) 12.24s

d) 13.7s

SOLUTION:

$$h(t) = -15\cos\left(\frac{\pi t}{30}\right) + 17$$

$$15 = -15\cos\left(\frac{\pi(t)}{30}\right) + 17$$

$$15 = -15\cos\left(\frac{\pi(t)}{30}\, x \left(\frac{\mathbf{180^0}}{\boldsymbol{\pi}}\right)\right) + 17$$

$$\boldsymbol{t_1 = 13.7s} \qquad \boldsymbol{t_2 = 46.3s}$$

PAST CE BOARD EXAM:

A Ferris wheel has a diameter of 30m with a center 17m above the ground. It makes one complete rotation every 60seconds. Assumed a rider starts at the lowest point. Which of the following equation best describe the height of the rider from the ground with respect to time.

TRIGONOMETRIC IDENTITIES

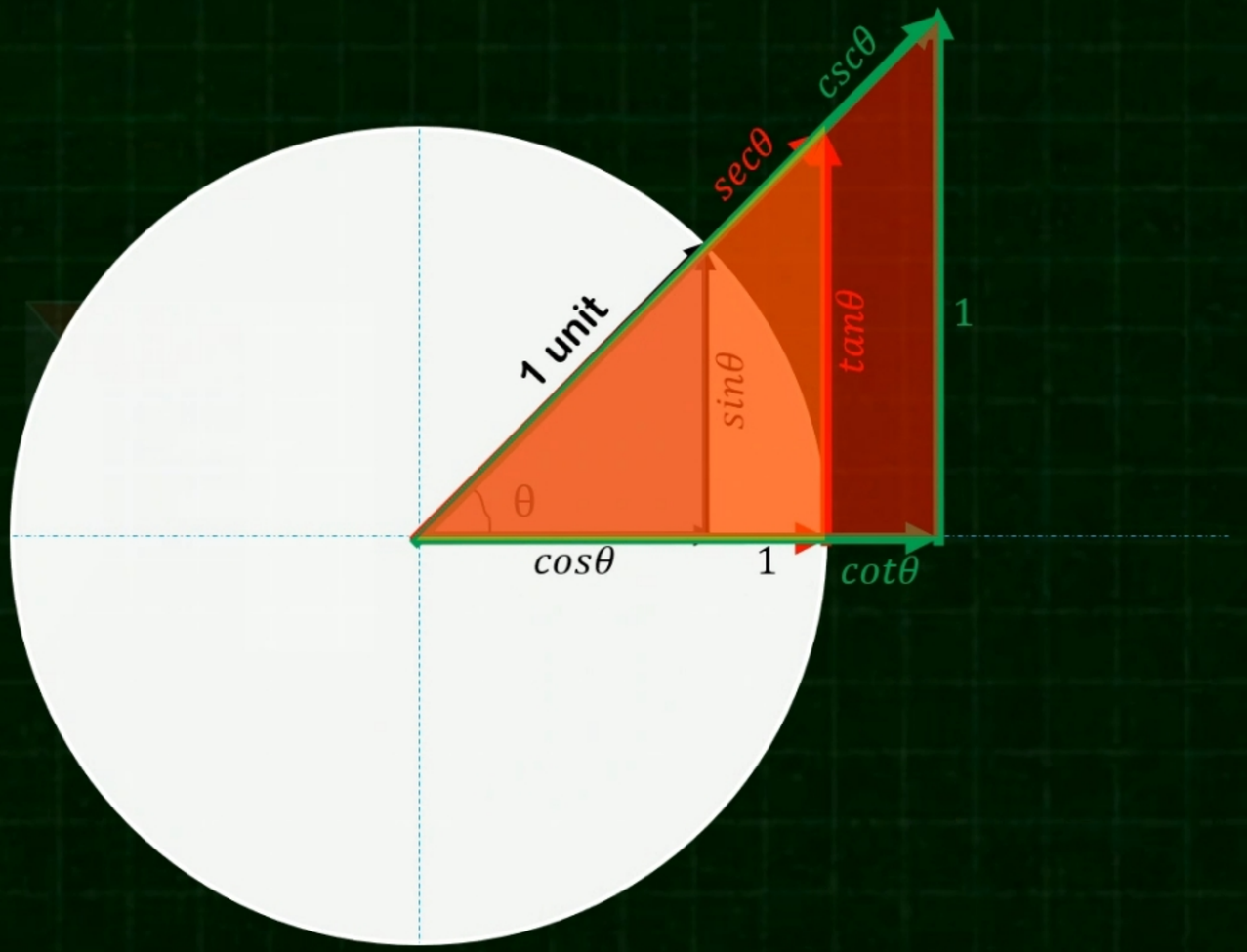
TRIGONOMETRIC IDENTITIES

PYTHAGOREAN IDENTITIES

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = tan^2 A + 1$$

$$\csc^2 A = \cot^2 A + 1$$



TRIGONOMETRIC IDENTITIES

SUM OF ANGLE IDENTITIES

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

AB: AB+BA

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

COS COS SIN SIN

AB: AB-AB

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

TRIGONOMETRIC IDENTITIES

DIFFERENCE OF ANGLE IDENTITIES

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

AB: AB+BA

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

COS COS SIN SIN

AB: AB-AB

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

DOUBLE ANGLE IDENTITIES

$$\sin(A + A) = \sin A \cos A + \sin A \cos A$$

AB: AB+BA

$$\boxed{\sin(2A) = 2 \sin A \cos A}$$

$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$

COS COS SIN SIN

AB: AB-AB

$$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$$

$$\tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

AB: +AB-AB

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

DOUBLE ANGLE IDENTITIES

$$\sin(A + A) = \sin A \cos A + \sin A \cos A$$

AB: AB+BA

$$\boxed{\sin(2A) = 2 \sin A \cos A}$$

$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$

COS COS SIN SIN

AB: AB-AB

$$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$$

$$\tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

AB: +AB-AB

$$\boxed{\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}}$$

DOUBLE ANGLE IDENTITIES

$$\cos(2A) = \cos^2 A - \sin^2 A$$

PYTHAGOREAN IDENTITIES

$$\sin^2 A + \cos^2 A = 1$$
$$\sin^2 A = 1 - \cos^2 A$$
$$\cos^2 A = 1 - \sin^2 A$$

$$\cos(2A) = \cos^2 A - (1 - \cos^2 A)$$
$$\cos(2A) = \cos^2 A - 1 + \cos^2 A$$
$$\boxed{\cos(2A) = 2\cos^2 A - 1}$$

$$\cos(2A) = 1 - \sin^2 A - \sin^2 A$$
$$\boxed{\cos(2A) = 1 - 2\sin^2 A}$$

PAST CE BOARD EXAM:

Sin (B – A) = _____when B = 270degrees and A is an acute angle.

a) Sin A
b) -Sin A
c) Cos A
d) -Cos A

SOLUTION:

Difference of angle formulas

$$Sin\,(270 - A) = Sin\,270\,Cos\,A - Sin\,A\,Cos\,270$$

$$Sin\,(270 - A) = \boxed{-Cos\,A}$$

Mode: 1

sin(270 − A) : sin A

: −sinA : cos A

: −cosA

CALC A? any values 35.789

=

sin(270 − A) =

a) sin A
b) − sin A
c) cos A
d) − cosA

PAST CE BOARD EXAM:

Sin (B – A) = _____when B = 270degrees and A is an acute angle.

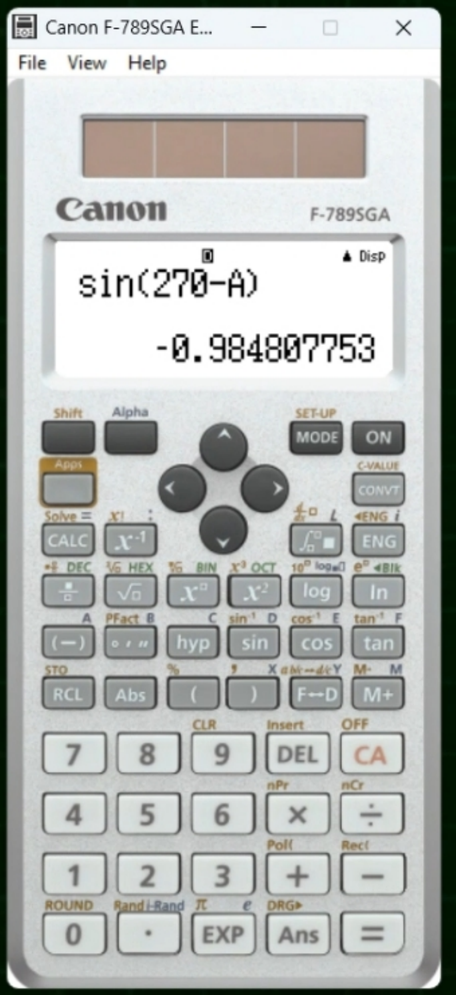
a) Sin A
b) -Sin A
c) Cos A
d) -Cos A

SOLUTION:

Difference of angle formulas

$$Sin\,(270 - A) = Sin\,270\,Cos\,A - Sin\,A\,Cos\,270$$

$$Sin\,(270 - A) = \boxed{-Cos\,A}$$



PAST CE BOARD EXAM:

Sin (B – A) = _____when B = 270degrees and A is an acute angle.

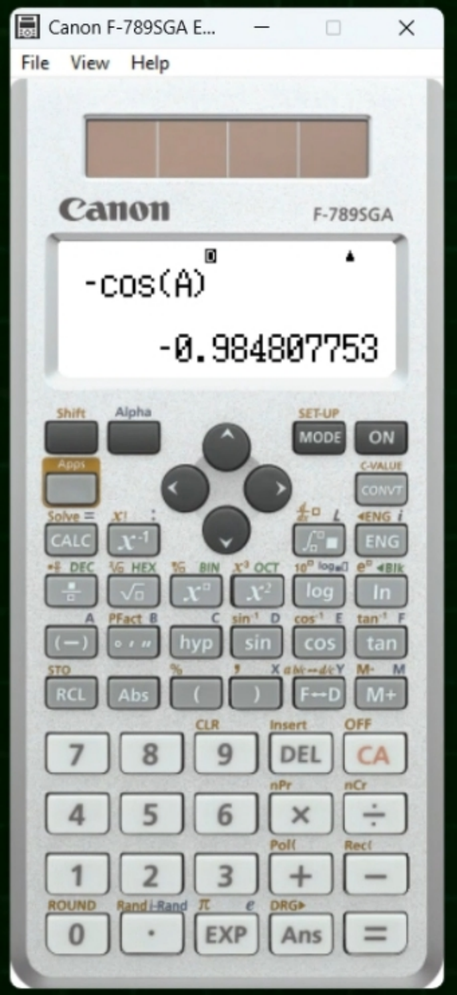
a) Sin A
b) -Sin A
c) Cos A
d) -Cos A

SOLUTION:

Difference of angle formulas

$$Sin\,(270 - A) = Sin\,270\,Cos\,A\; - Sin\,A\,Cos\,270$$

$$Sin\,(270 - A) = \boxed{-Cos\,A}$$



PAST CE BOARD EXAM:

Sin (B – A) = _____when B = 270degrees and A is an acute angle.

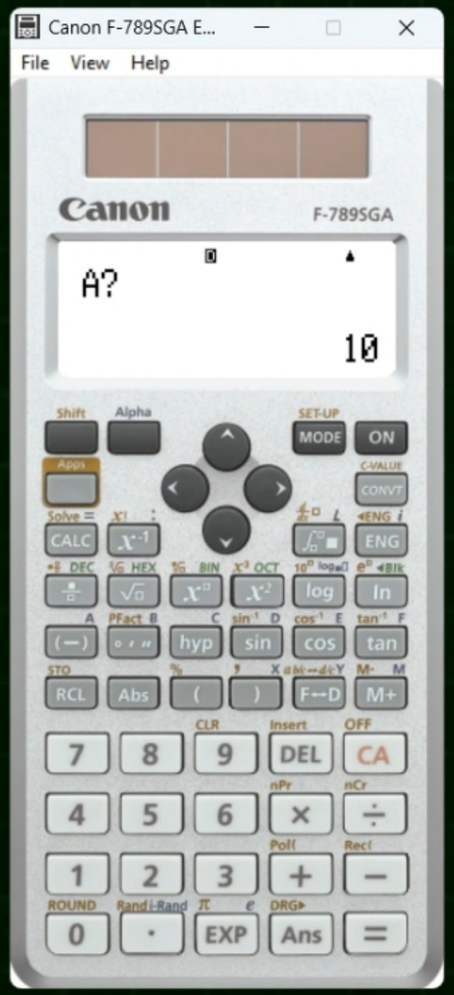
a) Sin A
b) -Sin A
c) Cos A
d) -Cos A

SOLUTION:

Difference of angle formulas

$$Sin\,(270 - A) = Sin\,270\,Cos\,A - Sin\,A\,Cos\,270$$

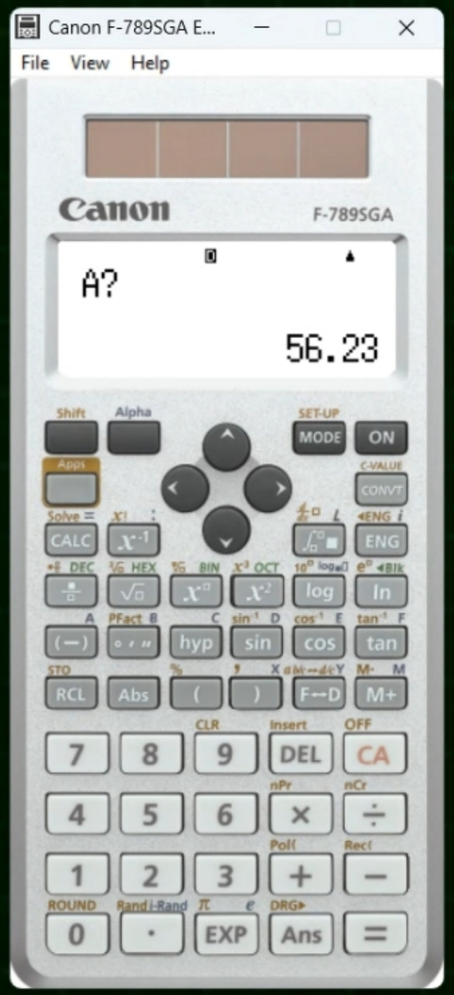
$$Sin\,(270 - A) = \boxed{-Cos\,A}$$



PAST CE BOARD EXAM:

$(\cos A)^4 - (\sin A)^4 =$ _____

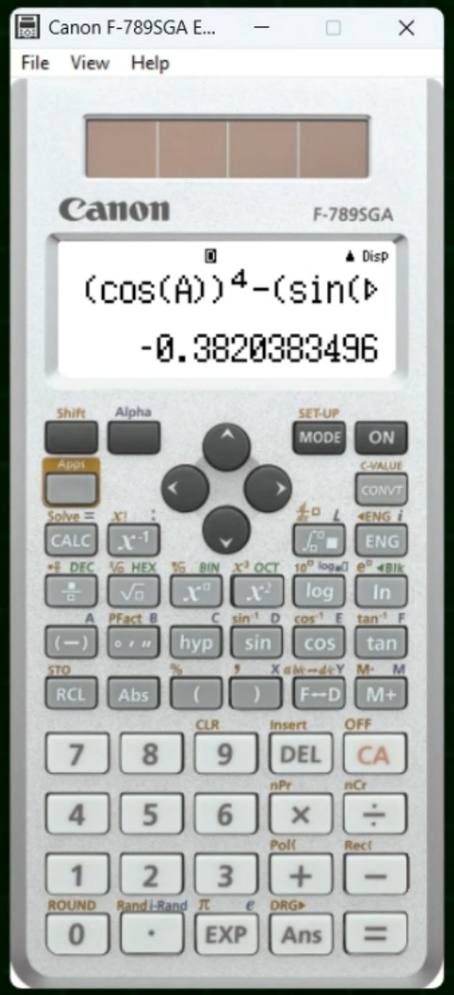
a) Sin 2A
b) Sin 4A
c) Cos 2A
d) Cos 4A



PAST CE BOARD EXAM:

$(\cos A)^4 - (\sin A)^4 =$ _____

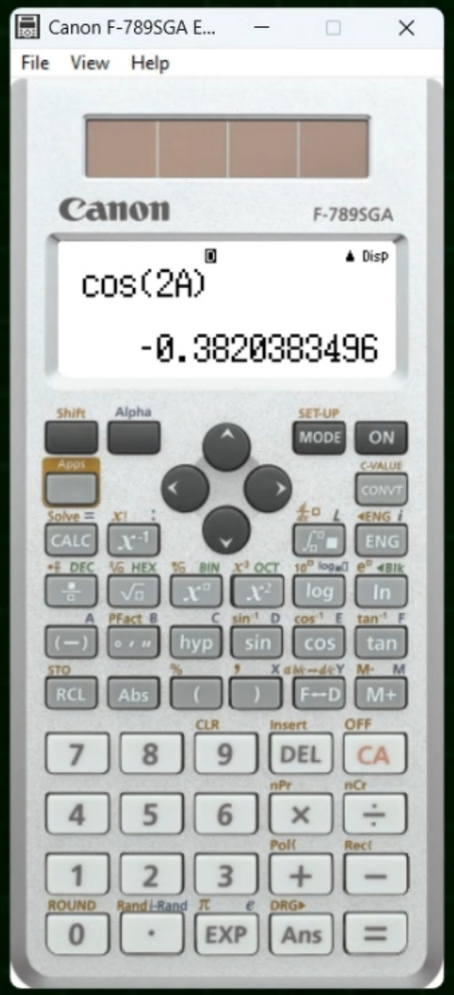
a) Sin 2A
b) Sin 4A
c) Cos 2A
d) Cos 4A



PAST CE BOARD EXAM:

$(\cos A)^4 - (\sin A)^4 =$ _____

a) Sin 2A
b) Sin 4A
c) Cos 2A
d) Cos 4A



PAST CE BOARD EXAM:

$(\cos A)^4 - (\sin A)^4 =$ _____

a) Sin 2A
b) Sin 4A
c) Cos 2A
d) Cos 4A

SOLUTION:

$(\cos A)^4 - (\sin A)^4$

$[(\cos A)^2 - (\sin A)^2][(\cos A)^2 + (\sin A)^2]$

$x^2 - y^2 = (x-y)(x+y)$

$x^4 - y^4$

$(x^2)^2 - (y^2)^2$

$(x^2 - y^2)(x^2 + y^2)$

PAST CE BOARD EXAM:

$(\cos A)^4 - (\sin A)^4 =$ _____

a) Sin 2A
b) Sin 4A
c) Cos 2A
d) Cos 4A

SOLUTION:

$$(\cos A)^4 - (\sin A)^4$$

$$[(\cos A)^2 - (\sin A)^2][(\cos A)^2 + (\sin A)^2]$$

Pythagorean Identities

$$(\cos A)^2 + (\sin A)^2 = 1$$

$$[(\cos A)^2 - (\sin A)^2](1)$$

By Double Angle Formulas:

$$[(\cos A)^2 - (\sin A)^2] = \boxed{\cos 2A}$$

PAST CE BOARD EXAM NOV.2005:

$A = 4\cos x - 6\sin x$

$B = 4\sin x + 6\cos x$

Find the value of $A^2 + B^2$

a) 54 b) 52 c) 53 d)50

PAST CE BOARD EXAM NOV.2005:

$A = 4\cos x - 6\sin x$

$B = 4\sin x + 6\cos x$

Find the value of $A^2 + B^2$

a) 54 b) 52 c) 53 d)50

SOLUTION:

$A^2 + B^2$

$(4\cos x - 6\sin x)^2 + (4\sin x + 6\cos x)^2$

$(4\cos x - 6\sin x)^2 + (4\sin x + 6\cos x)^2$

$[16\cos^2 x - 48\,\text{sinx cosx} + 36\sin^2\text{x}] + [16\sin^2 x + 48\,\text{sinx cosx} +$

$[16\cos^2 x - 48\,\text{sinx cosx} + 36\sin^2\text{x}] + [16\sin^2 x + 48\,\text{sinx cosx} +$

$[16\cos^2 x + 36\sin^2\text{x}] + [16\sin^2 x + 36\cos^2 x]$

$[16\sin^2 x + 36\sin^2\text{x}] + [16\cos^2 x + 36\cos^2 x]$

$[52\sin^2\text{x}] + [52\cos^2 x]$

$52(\sin^2 x + \cos^2 x)$ *where*: $\sin^2 x + \cos^2 x = 1$

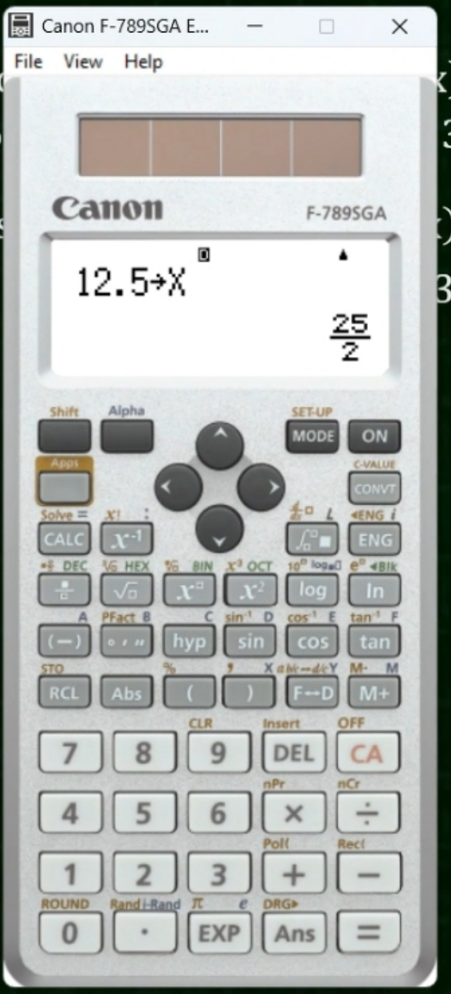
$\boxed{52(1) = 52}$

$(4\cos x - 6\sin x)^2 = (4$ $) + (6\text{sinx})^2$

$(4\cos x - 6\sin x)^2 = 16$ $36\sin^2\text{x}$

$(4\sin x + 6\cos x)^2 = (4$ $) + (6\text{cosx})^2$

$(4\sin x + 6\cos x)^2 = 16$ $36\cos^2 x$



PAST CE BOARD EXAM NOV.2005:

$A = 4\cos x - 6\sin x$

$B = 4\sin x + 6\cos x$

Find the value of $A^2 + B^2$

a) 54 b) 52 c) 53 d)50

SOLUTION:

$A^2 + B^2$

$(4\cos x - 6\sin x)^2 + (4\sin x + 6\cos x)^2$

$(4\cos x - 6\sin x)^2 + (4\sin x + 6\cos x)^2$

$[16\cos^2 x - 48\,\text{sinx cosx} + 36\sin^2\text{x}] + [16\sin^2 x + 48\,\text{sinx cosx} +$

$[16\cos^2 x - 48\,\text{sinx cosx} + 36\sin^2\text{x}] + [16\sin^2 x + 48\,\text{sinx cosx} +$

$[16\cos^2 x + 36\sin^2\text{x}] + [16\sin^2 x + 36\cos^2 x]$

$[16\sin^2 x + 36\sin^2\text{x}] + [16\cos^2 x + 36\cos^2 x]$

$[52\sin^2\text{x}] + [52\cos^2 x]$

$52(\sin^2 x + \cos^2 x)$ *where*: $\sin^2 x + \cos^2 x = 1$

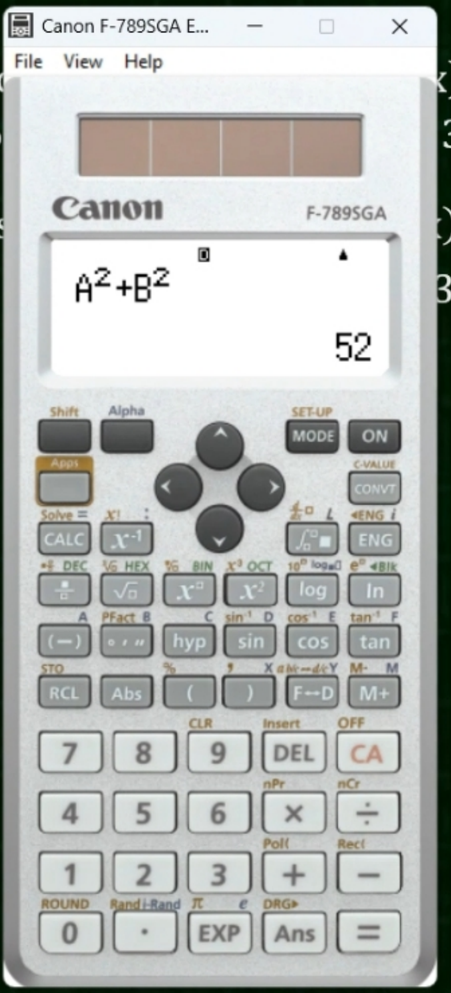
$\boxed{52(1) = 52}$

$(4\cos x - 6\sin x)^2 = (4$ $) + (6\text{sinx})^2$

$(4\cos x - 6\sin x)^2 = 16$ $36\sin^2\text{x}$

$(4\sin x + 6\cos x)^2 = (4$ $) + (6\text{cosx})^2$

$(4\sin x + 6\cos x)^2 = 16$ $36\cos^2 x$



PAST CE BOARD EXAM NOV.2005:

$A = 4\cos x - 6\sin x$

$B = 4\sin x + 6\cos x$

Find the value of $A^2 + B^2$

a) 54 b) 52 c) 53 d)50

SOLUTION:

$A^2 + B^2$

$(4\cos x - 6\sin x)^2 + (4\sin x + 6\cos x)^2$

$(4\cos x - 6\sin x)^2 + (4\sin x + 6\cos x)^2$

$[16\cos^2 x - 48\,\text{sinx cosx} + 36\sin^2\text{x}] + [16\sin^2 x + 48\,\text{sinx cosx} + 36\cos^2 x]$

$[16\cos^2 x - 48\,\text{sinx cosx} + 36\sin^2\text{x}] + [16\sin^2 x + 48\,\text{sinx cosx} + 36\cos^2 x]$

$[16\cos^2 x + 36\sin^2\text{x}] + [16\sin^2 x + 36\cos^2 x]$

$[16\sin^2 x + 36\sin^2\text{x}] + [16\cos^2 x + 36\cos^2 x]$

$[52\sin^2\text{x}] + [52\cos^2 x]$

$52(\sin^2 x + \cos^2 x)$ *where*: $\sin^2 x + \cos^2 x = 1$

$\boxed{52(1) = 52}$

$(4\cos x - 6\sin x)^2 = (4\cos\text{x})^2 - 2(4\cos\text{x})(6\sin\text{x}) + (6\sin\text{x})^2$

$(4\cos x - 6\sin x)^2 = 16\cos^2 x - 48\,\text{sinx cosx} + 36\sin^2\text{x}$

$(4\sin x + 6\cos x)^2 = (4\sin\text{x})^2 + 2(4\sin\text{x})(6\cos\text{x}) + (6\cos\text{x})^2$

$(4\sin x + 6\cos x)^2 = 16\sin^2 x + 48\,\text{sinx cosx} + 36\cos^2 x$

TEST YOURSELF:

$sin3A =$ _____?

a) $\sin A\,[1 + 2\cos 2A]$

b) $\sin A\,[1 + 3\cos 2A]$

c) $\sin A\,[1 + 4\cos 2A]$

d) $\sin A\,[1 + 5\cos 2A]$

SOLUTION:

$\sin 3A = \sin(2A + A)$

Sum of angle formulas

$Sin\,(2A + A) = Sin\,2A\,Cos\,A + Sin\,A\,Cos\,2A$

By Double Angle Formulas:

$Sin\,(2A) = 2Sin\,A\,Cos\,A$

$\mathrm{Cos}\,(2A) = 1 - 2Sin^2\,A$

$Sin\,(2A + A) = (2Sin\,A\,Cos\,A)\,Cos\,A + Sin\,A(1 - 2Sin^2\,A)$

$Sin\,(2A + A) = (2Sin\,A)Cos^2\,A + Sin\,A - 2Sin^3\,A)$

$Sin\,(2A + A) = \sin A\,[2Cos^2\,A + 1 - 2Sin^2\,A]$

$Sin\,(2A + A) = \sin A\,[1 + 2Cos^2\,A - 2Sin^2\,A]$

$Sin\,(2A + A) = \sin A\,[1 + 2(Cos^2\,A - Sin^2\,A)]$

By Double Angle Formulas:

$[(\cos A)^2 - (\sin A)^2] = \cos 2A$

$Sin\,(2A + A) = \boxed{\sin A\,[1 + 2\cos 2A]}$

TEST YOURSELF:

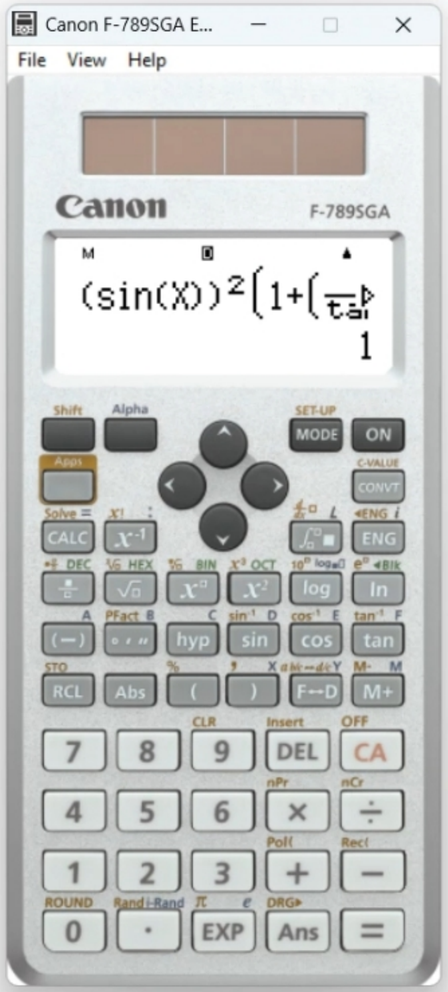
$\sin^2 x\left(1+\cot^2 x\right) =$ _____.

a) $\sin^2 x$

b) $\sec^2 x$

c) $\sec^2 x \sin^2 x$

d) 1



TEST YOURSELF:

$\sin^2 x\left(1+\cot^2 x\right) =$ _____.

a) $\sin^2 x$

b) $\sec^2 x$

c) $\sec^2 x \sin^2 x$

d) 1

SOLUTION:

$$\sin^2 x\,(1+\cot^2 x)$$

$$\sin^2 x + \sin^2 x \cot^2 x$$

$$\sin^2 x + \sin^2 x\left(\frac{1}{\tan^2 x}\right)$$

$$\sin^2 x + \sin^2 x\left(\frac{1}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}\right)$$

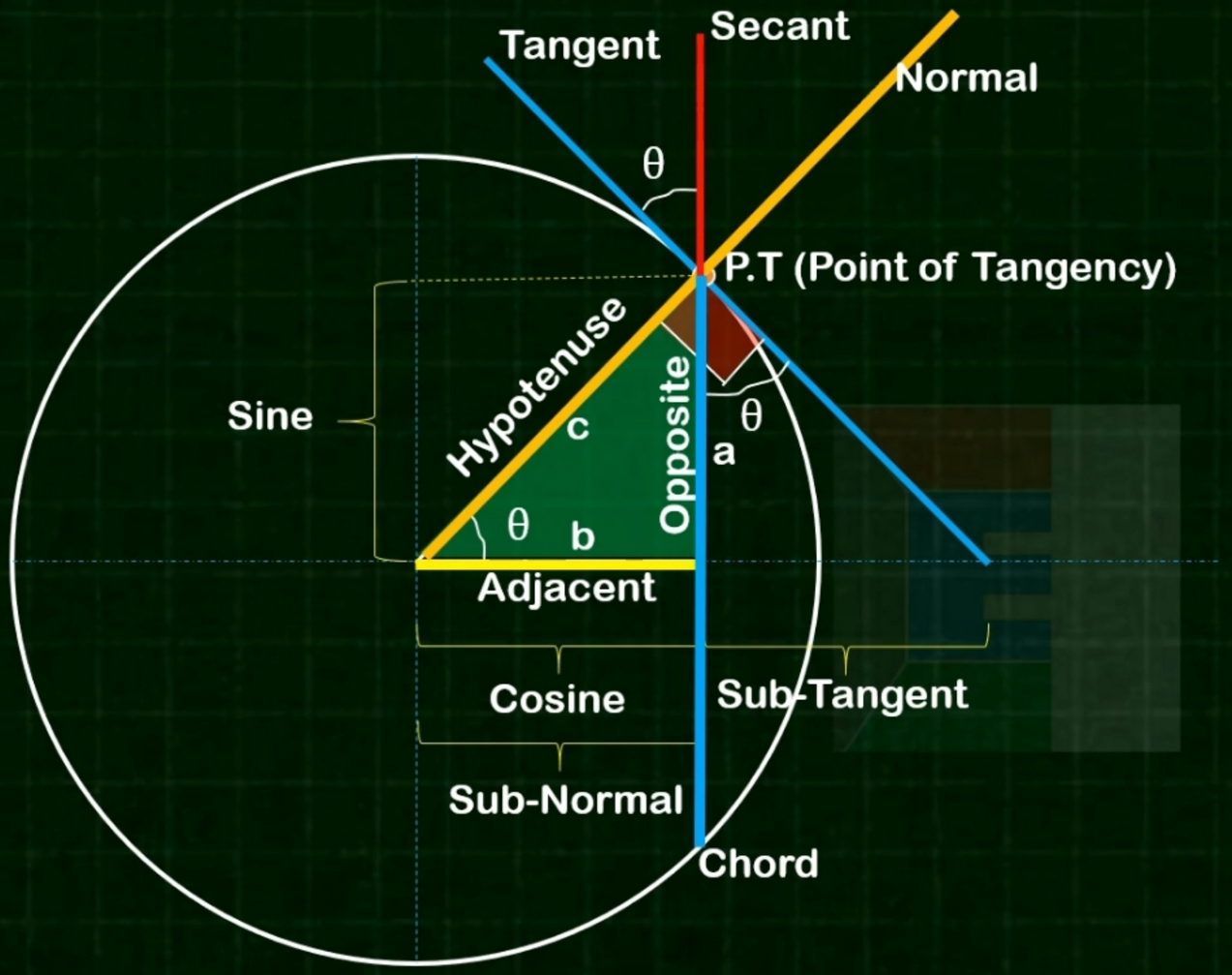
$$\sin^2 x + \sin^2 x\left(\frac{\cos^2 x}{\sin^2 x}\right)$$

$$\sin^2 x + \cancel{\sin^2 x}\left(\frac{\cos^2 x}{\cancel{\sin^2 x}}\right)$$

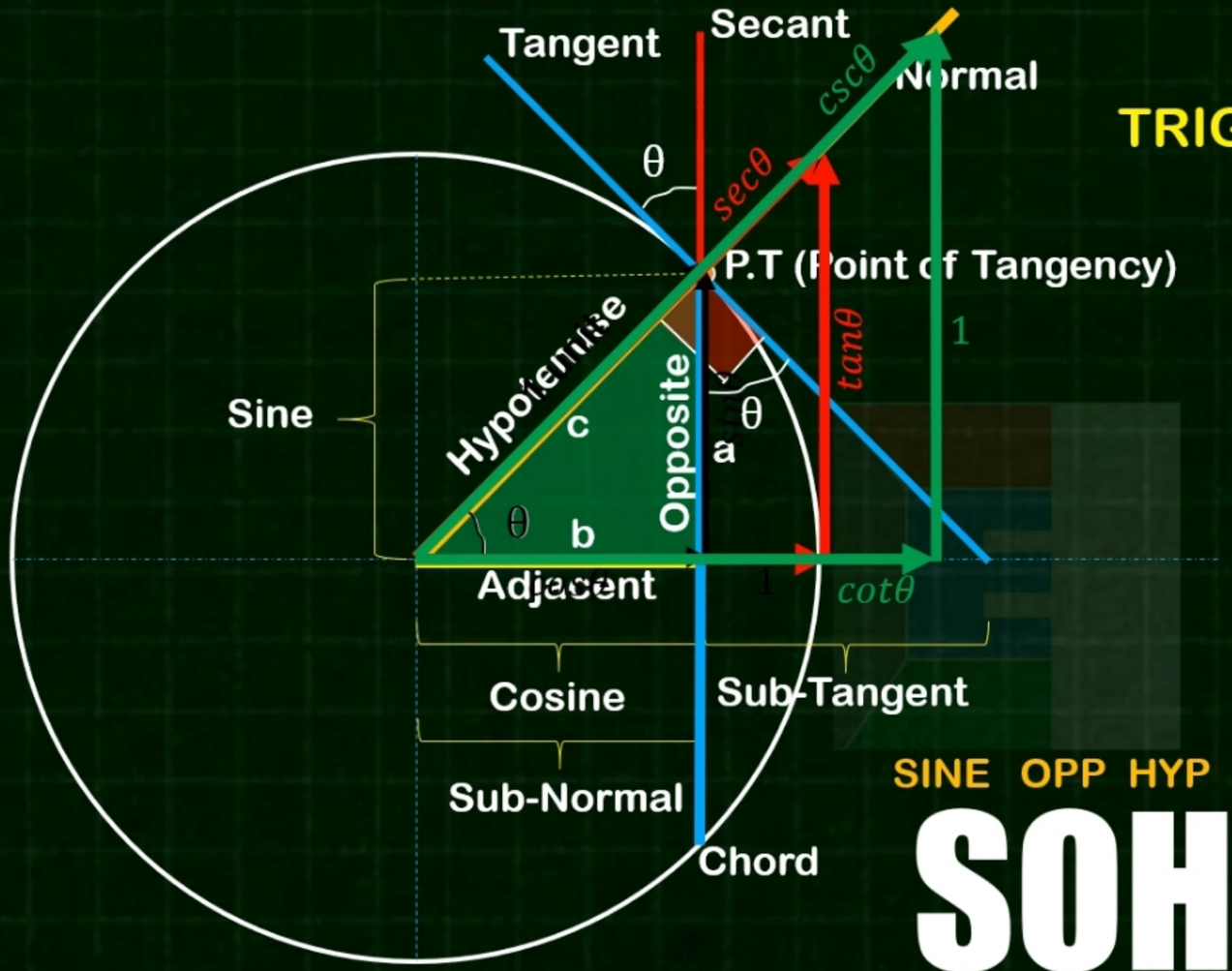
$$\sin^2 x + \cos^2 x$$

$$\boxed{1}$$

TRIGONOMETRIC FUNCTIONS


Tangent
Secant
Normal
θ
P.T (Point of Tangency)
Hypotenuse
c
Opposite
a
θ
Sine
θ
b
Adjacent
Cosine
Sub-Tangent
Sub-Normal
Chord

RIGHT TRIANGLE TRIGONOMETRIC FUNCTIONS



$$\sin\theta = \frac{OPP}{HYP} = \frac{a}{c}$$

$$\cos\theta = \frac{ADJ}{HYP} = \frac{b}{c}$$

$$\tan\theta = \frac{OPP}{ADJ} = \frac{a}{b}$$

Cosine Function

SINE OPP HYP

TAN OPP ADJ

SOH CAH TOA

COS ADJ HYP

Sine Function

Tangent Function

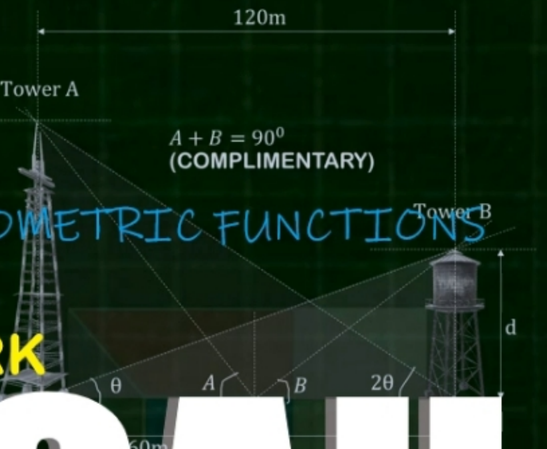
FRIENDLY REMINDER FROM TRIGONOMETRIC FUNCTIONS

STUDYING OUR HOMEWORK

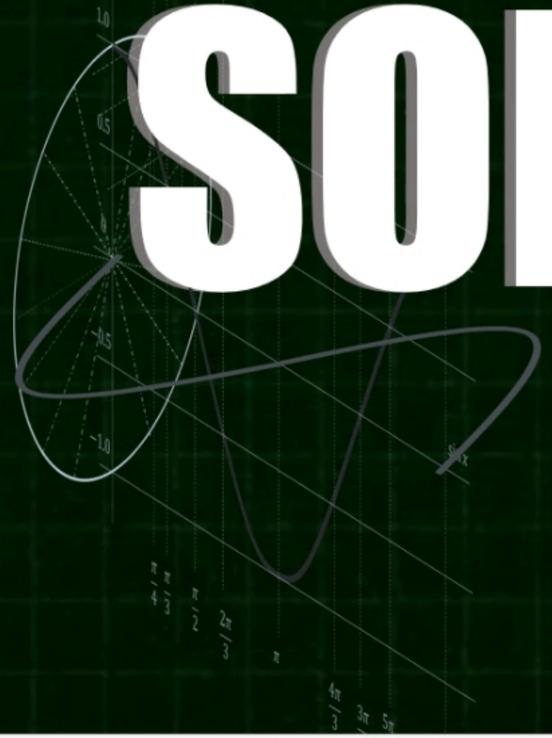
TO OBTAIN ACHIEVEMENT

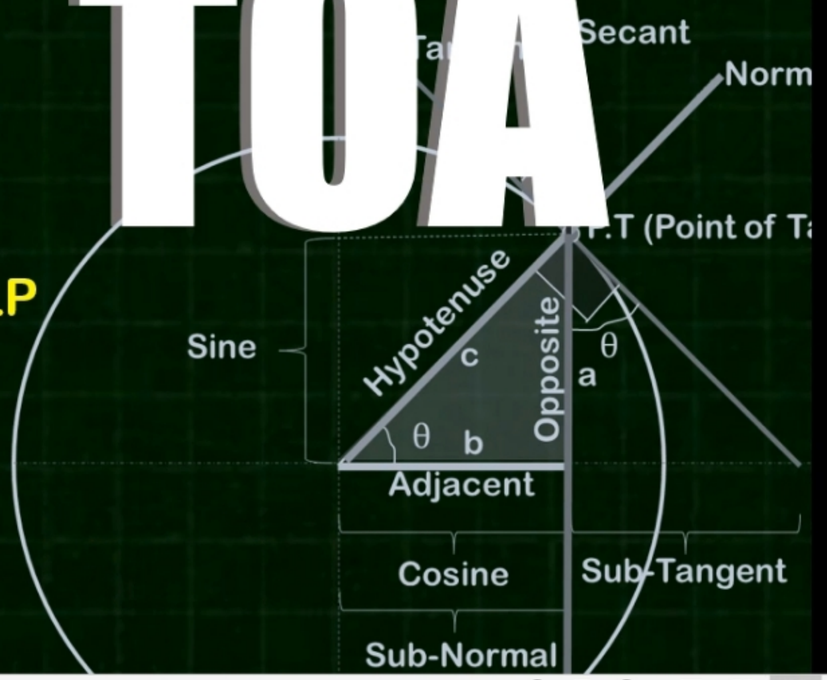
SOH CAH TOA

CAN ALWAYS HELP



$A + B = 90^0$
(COMPLIMENTARY)





TRIGONOMETRIC FUNCTIONS

$$\sin\theta = \frac{OPP}{HYP} = \frac{a}{c}$$ **SINE**

$$\cos\theta = \frac{ADJ}{HYP} = \frac{b}{c}$$ **COSINE**

$$\tan\theta = \frac{OPP}{ADJ} = \frac{a}{b}$$ **TANGENT**

Cosecant

$$\csc\theta = \frac{HYP}{OPP} = \frac{c}{a}$$
$$\csc\theta = 1/\sin\theta$$

Secant

$$\sec\theta = \frac{HYP}{ADJ} = \frac{c}{b}$$
$$\sec\theta = 1/\cos\theta$$

CO - tangent

$$\cot\theta = \frac{ADJ}{OPP} = \frac{b}{a}$$
$$\cot\theta = 1/\tan\theta$$

Secant Function

CSC HYP OPP — COT ADJ OPP

CHO SHA CAO

SEC HYP ADJ

Cosecant Function — Cotangent Function

TRIGONOMETRIC FUNCTIONS

$$\sin\theta = \frac{OPP}{HYP} = \frac{a}{c}$$

SINE

$$\cos\theta = \frac{ADJ}{HYP} = \frac{b}{c}$$

COSINE

$$\tan\theta = \frac{OPP}{ADJ} = \frac{a}{b}$$

TANGENT

Cosecant

$$\csc\theta = \frac{HYP}{OPP} = \frac{c}{a}$$
$$\csc\theta = 1/\sin\theta$$

Secant

$$\sec\theta = \frac{HYP}{ADJ} = \frac{c}{b}$$
$$\sec\theta = 1/\cos\theta$$

CO - tangent

$$\cot\theta = \frac{ADJ}{OPP} = \frac{b}{a}$$
$$\cot\theta = 1/\tan\theta$$

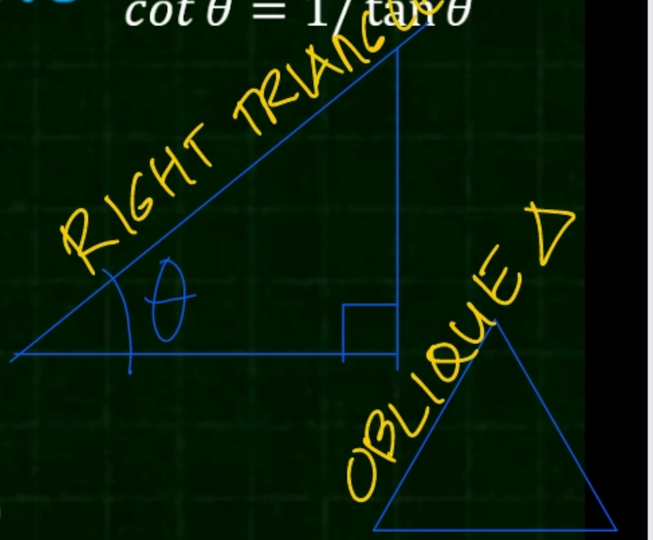
Secant Function

CSC HYP OPP — COT ADJ OPP

CHO SHA CAO

SEC HYP ADJ

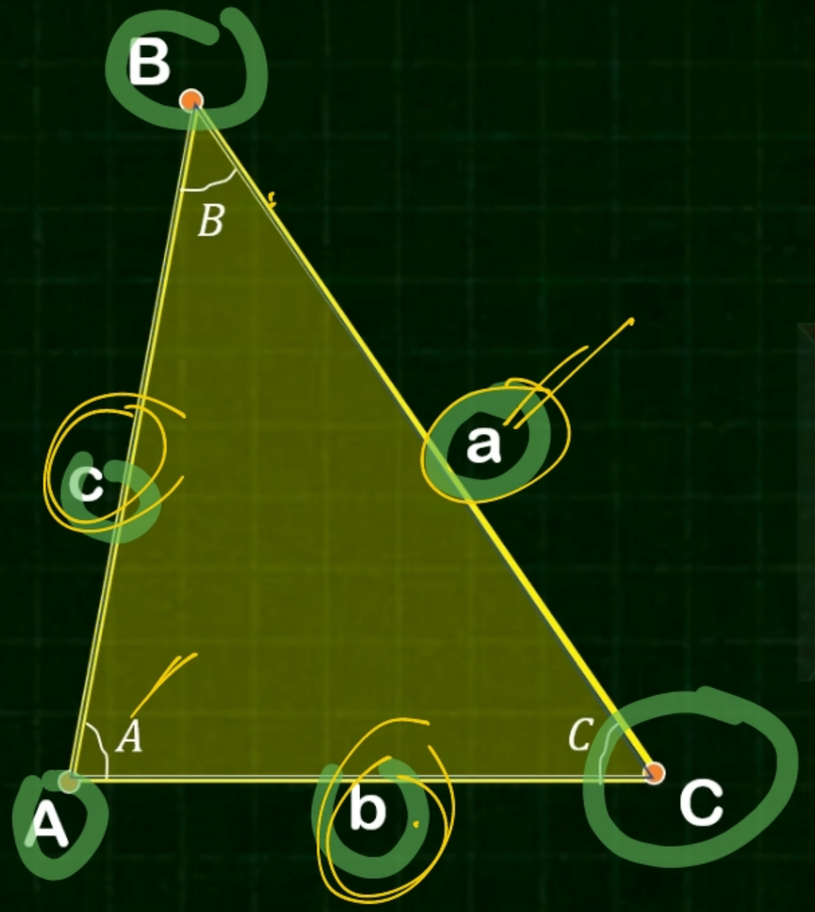
Cosecant Function — Cotangent Function



SHOOT YOUR OWN TRIGODANCE

SIN COS TAN

TrigoDance



OBLIQUE TRIANGLE TRIGONOMETRIC LAWS

SINE LAW:

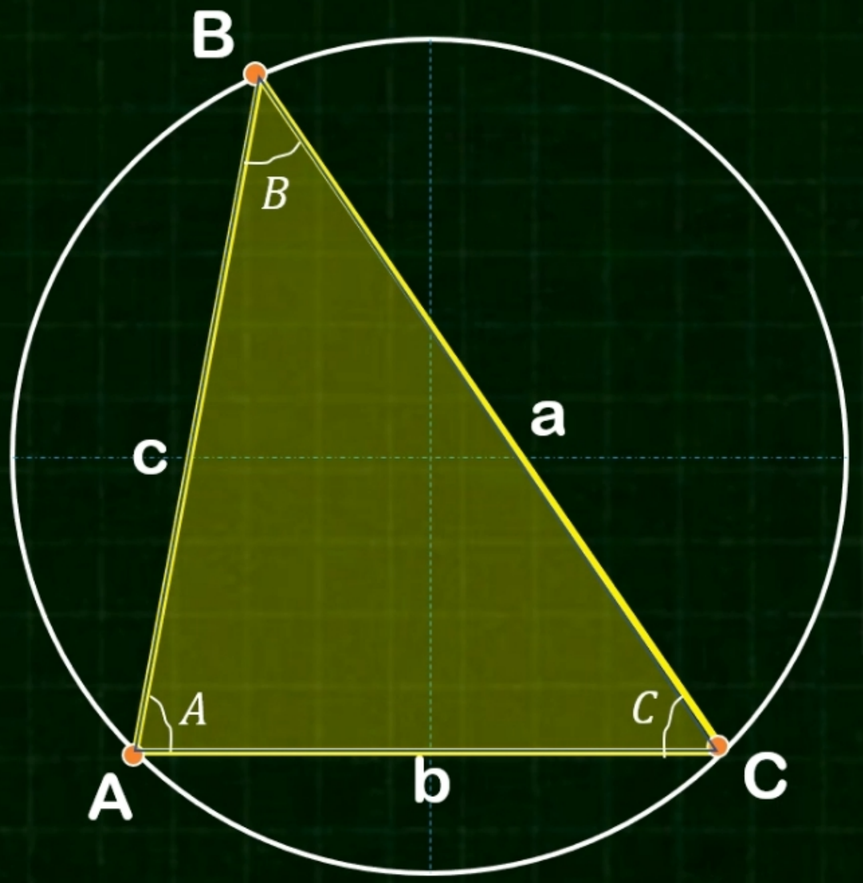
$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

COSINE LAW:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



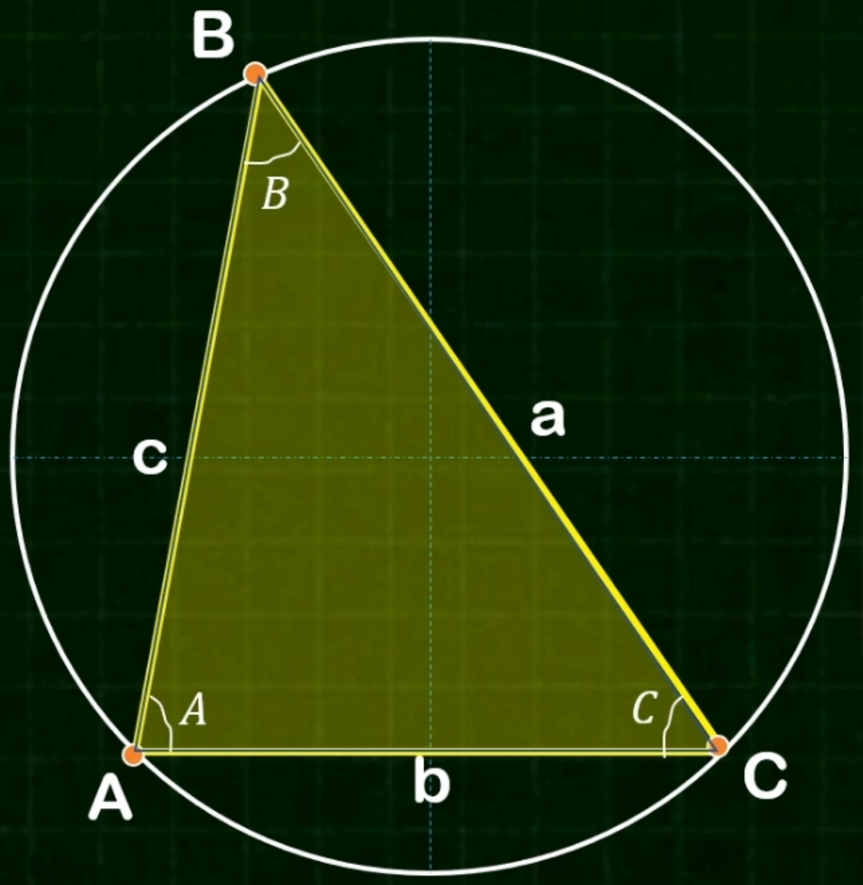
OBLIQUE TRIANGLE

TANGENT LAW:

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(B-C)}{\tan\frac{1}{2}(B+C)}$$

$$\frac{c-a}{c+a}=\frac{\tan\frac{1}{2}(C-A)}{\tan\frac{1}{2}(C-B)}$$



OBLIQUE TRIANGLE

MOLLWEIDE'S LAW:

$$\frac{a-b}{c}=\frac{\sin\frac{1}{2}(A-B)}{\cos\frac{1}{2}(C)}$$

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(A+B)}{\sin\frac{1}{2}(C)}$$

PROBLEM:

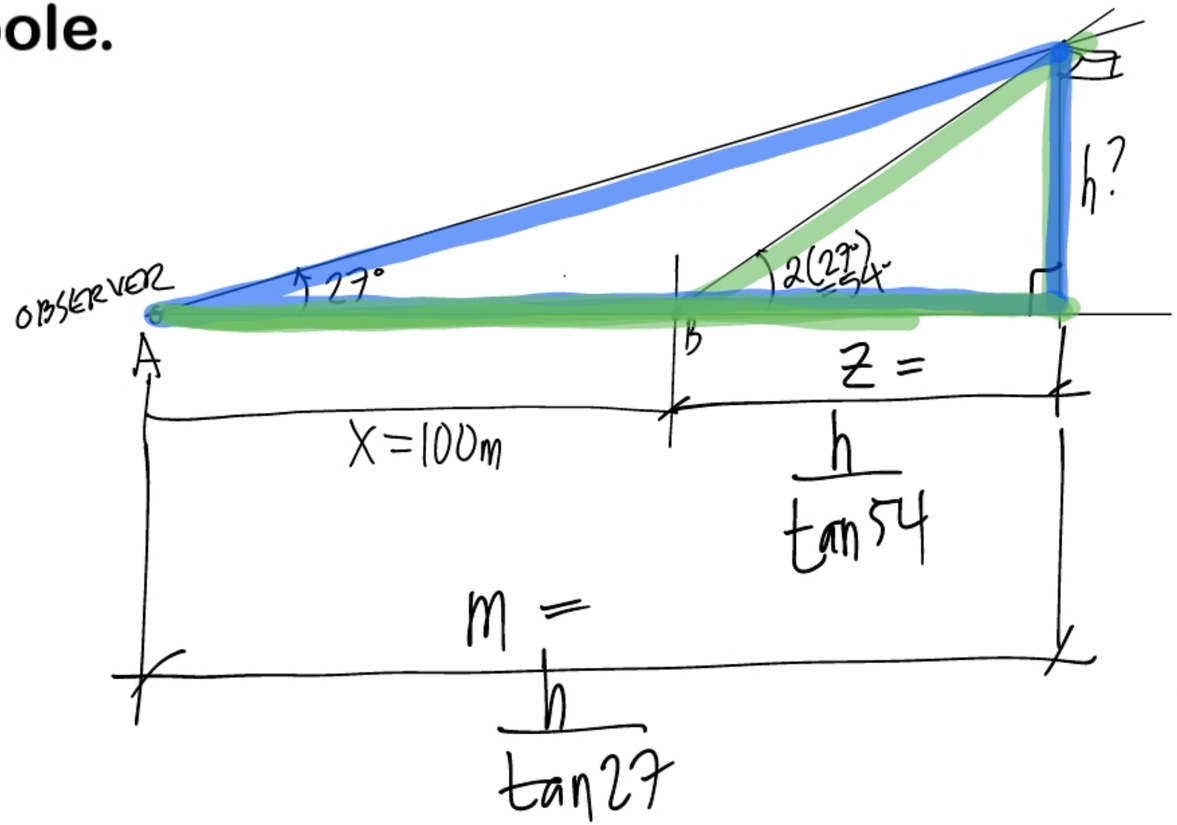
An observer at point A observed the angle of elevation of 27degrees to the top of the pole. If he move horizontally a distance x = 100m towards B, nearer to the pole, the angle of elevation of the pole at B is double that of A. Find the height of the pole.

a) 61m

b) 71m

c) 81m

d) 91m



$\tan 54 = \frac{h}{z}$

$z = \frac{h}{\tan 54}$

$\tan 27 = \frac{h}{m}$

$m = \frac{h}{\tan 27}$

$\frac{h}{\tan 27} = 100 + \frac{h}{\tan 54}$

$\therefore h = 80.902\text{m}$

PROBLEM:

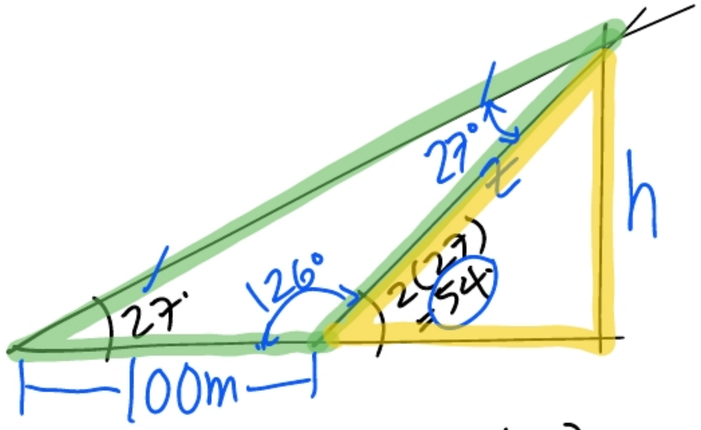
An observer at point A observed the angle of elevation of 27degrees to the top of the pole. If he move horizontally a distance x = 100m towards B, nearer to the pole, the angle of elevation of the pole at B is double that of A. Find the height of the pole.

a) 61m

b) 71m

c) 81m

d) 91m



SINE LAW: $\frac{100}{\sin 27} = \frac{z}{\sin(27)}$

$\therefore z = 100$

$h = z \sin 54$

$h = 100 \sin 54°$

$= 80.902\ m$

PROBLEM:

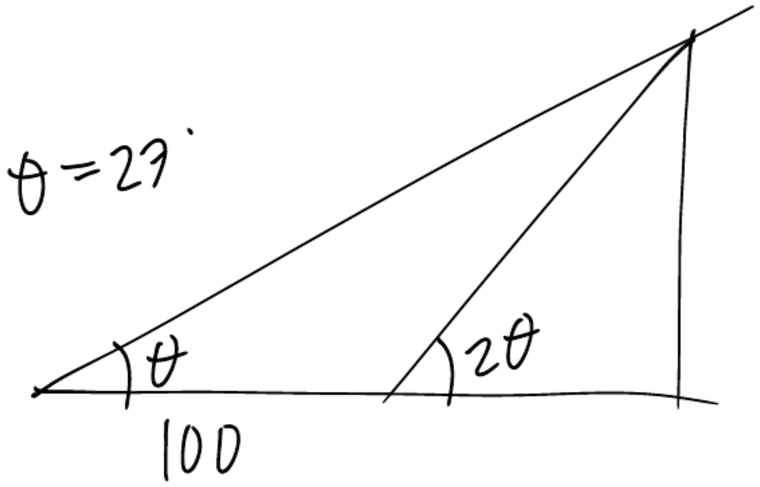
An observer at point A observed the angle of elevation of 27degrees to the top of the pole. If he move horizontally a distance x = 100m towards B, nearer to the pole, the angle of elevation of the pole at B is double that of A. Find the height of the pole.

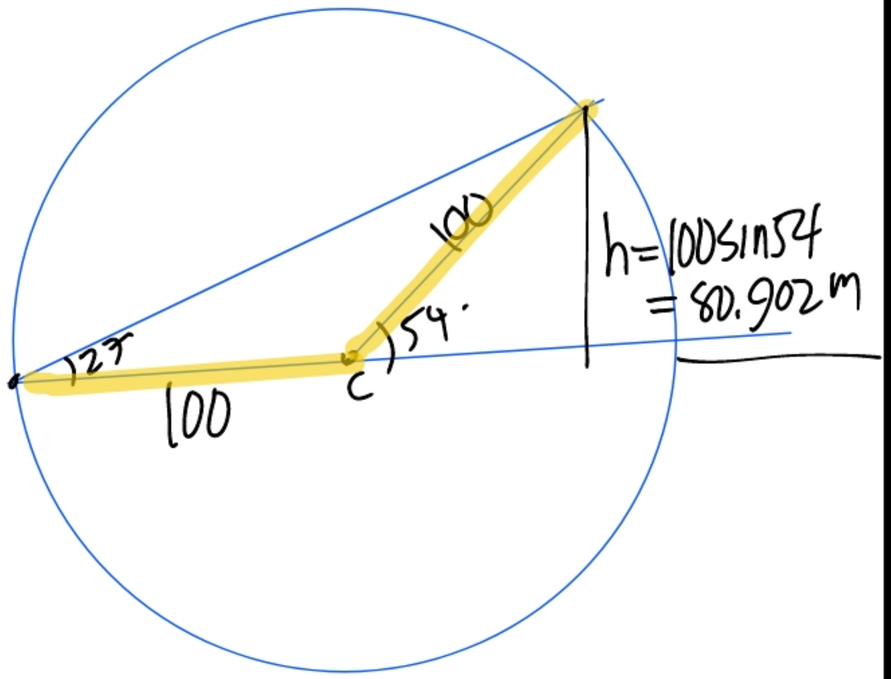
a) 61m

b) 71m

c) 81m

d) 91m





PROBLEM:

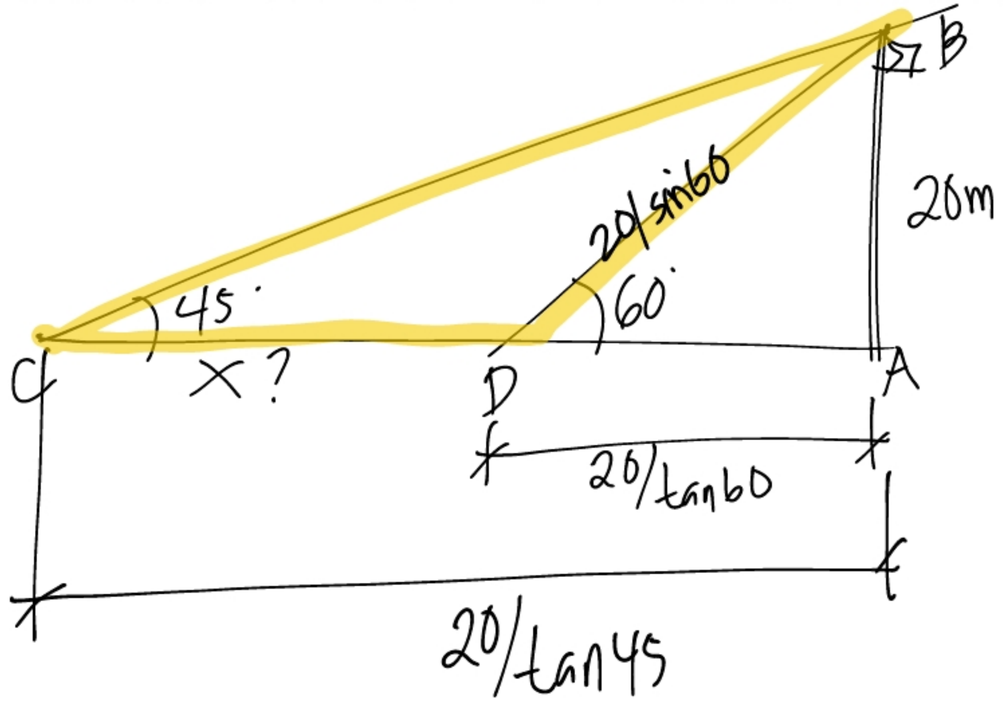
A man stands at C at a certain distance from a flagpole AB, which is 20m high. The angle of elevation of the top of AB at C is 45degrees. The man then walks towards the pole at D. The angle of elevation of the top of the pole measured from D is 60degrees. Find the distance he had walked.

a) 5.25m

b) 8.45m

c) 7.05m

d) 10.5m



$$\frac{20}{\tan 45} = X + \frac{20}{\tan 60}$$

$$X = 8.45\text{m}$$

CELE MAY 2015:

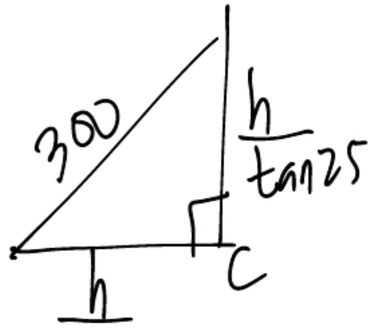
The angle of elevation of the top point D of a tower from A is 25degrees. From another point B, the angle of elevation of the top of the tower at D is 56degrees. Points A and B is 300m apart and are the same horizontal planes as the foot C of the tower. The horizontal angle subtended by A and B at the foot of the tower is 90degrees, find the height of the tower in meters.

a) 133.45m

b) 206.32m

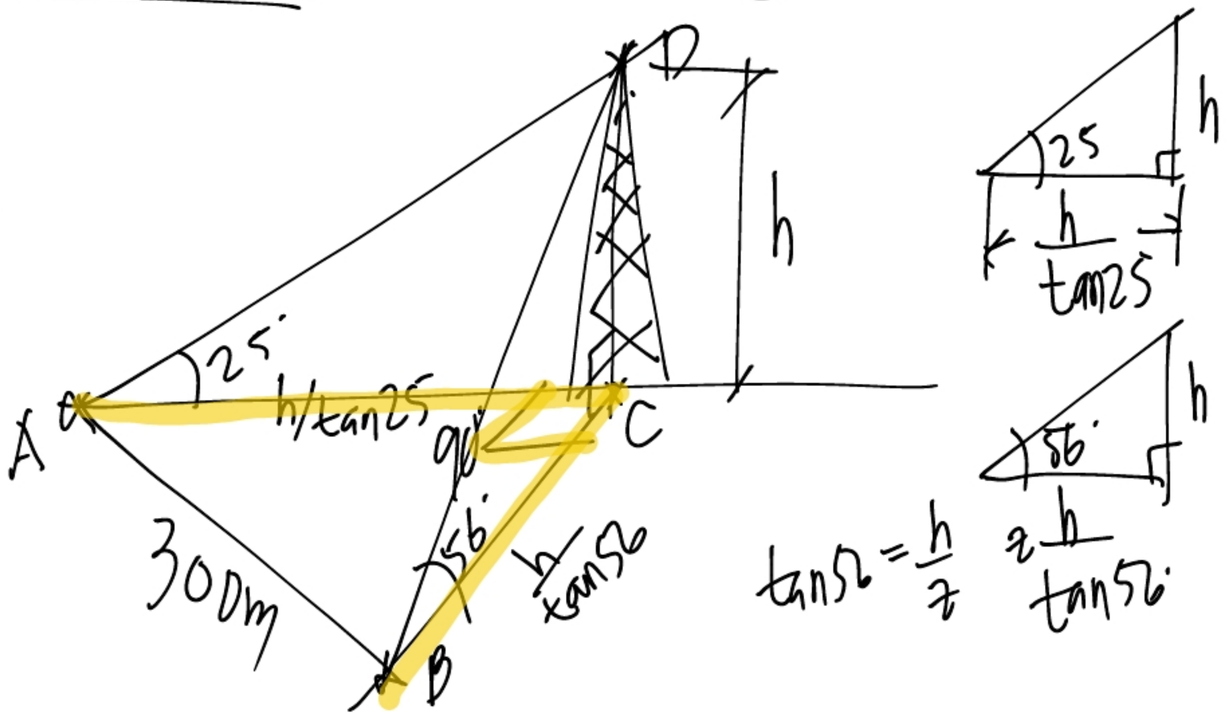
c) 365.11m

d) 122.76m



$$300^2 = \left(\frac{h}{\tan 56}\right)^2 + \left(\frac{h}{\tan 25}\right)^2$$

$$h = 133.45\text{m}$$



$$\tan 56 = \frac{h}{z}$$

PLANE TRIGONOMETRY:

Past CE Board Exam:

A geodetic engineer wants to determine the height of the tower by observing a vertical angle at a certain point. He then moves a horizontal distance of 60m towards the tower at the same vertical plane with the tower and observes the vertical angle of the tower to be double that of the first. He resume by moving further towards the tower at the same vertical plane with the tower at a distance of 20m and observes that the vertical angle of the tower is the compliment of the first angle. Determine the height of the tower. Assume that the different points of observations and the base of the tower lies on the same horizontal plane.

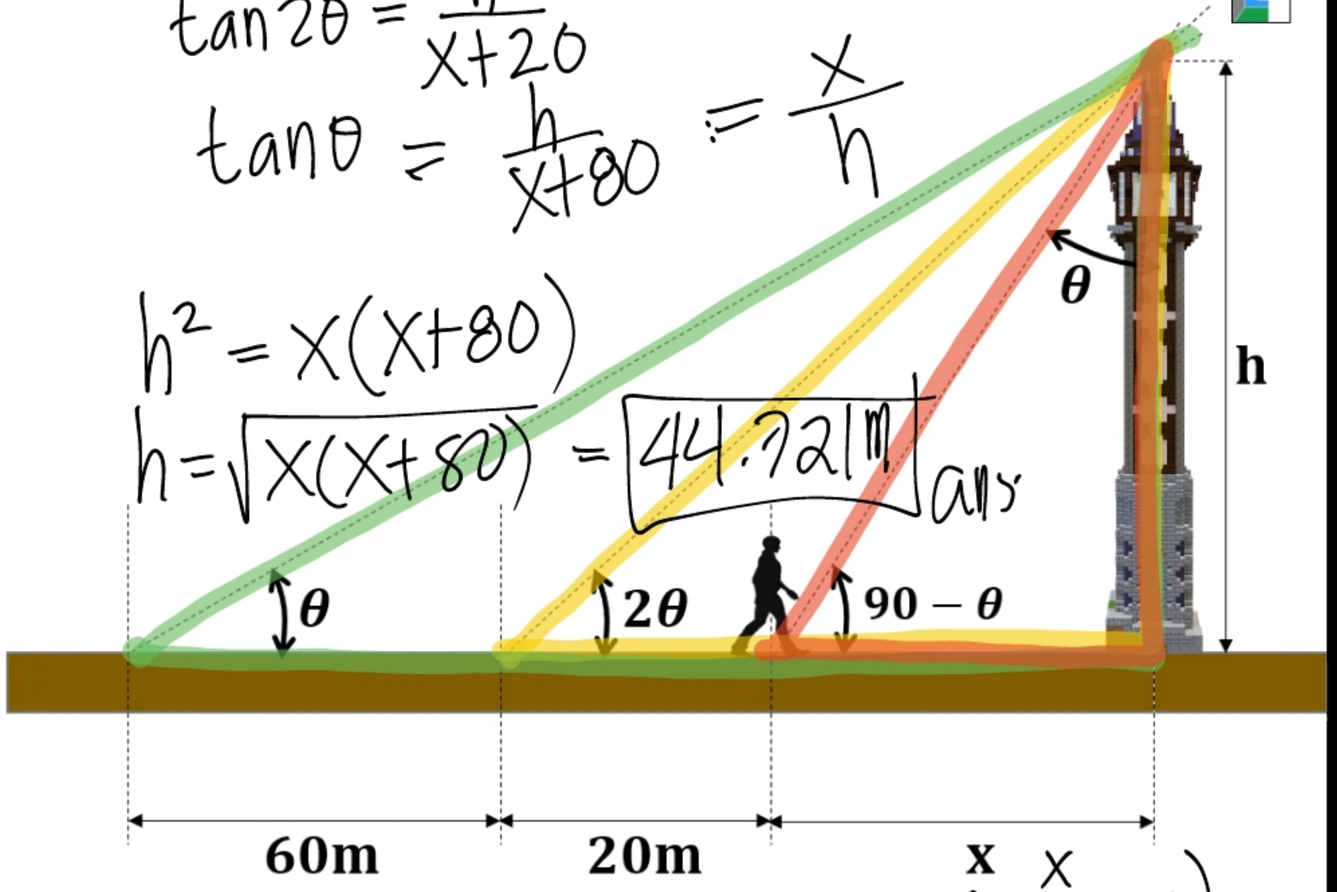
a) 32.952m b) 44.285m
c) 317.157m d) 44.720m

$\tan 2\theta = \frac{h}{x+20}$

$\tan\theta = \frac{h}{x+80} = \frac{x}{h}$

$h^2 = x(x+80)$

$h = \sqrt{x(x+80)} = \boxed{44.721\text{m}}$ ans



$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

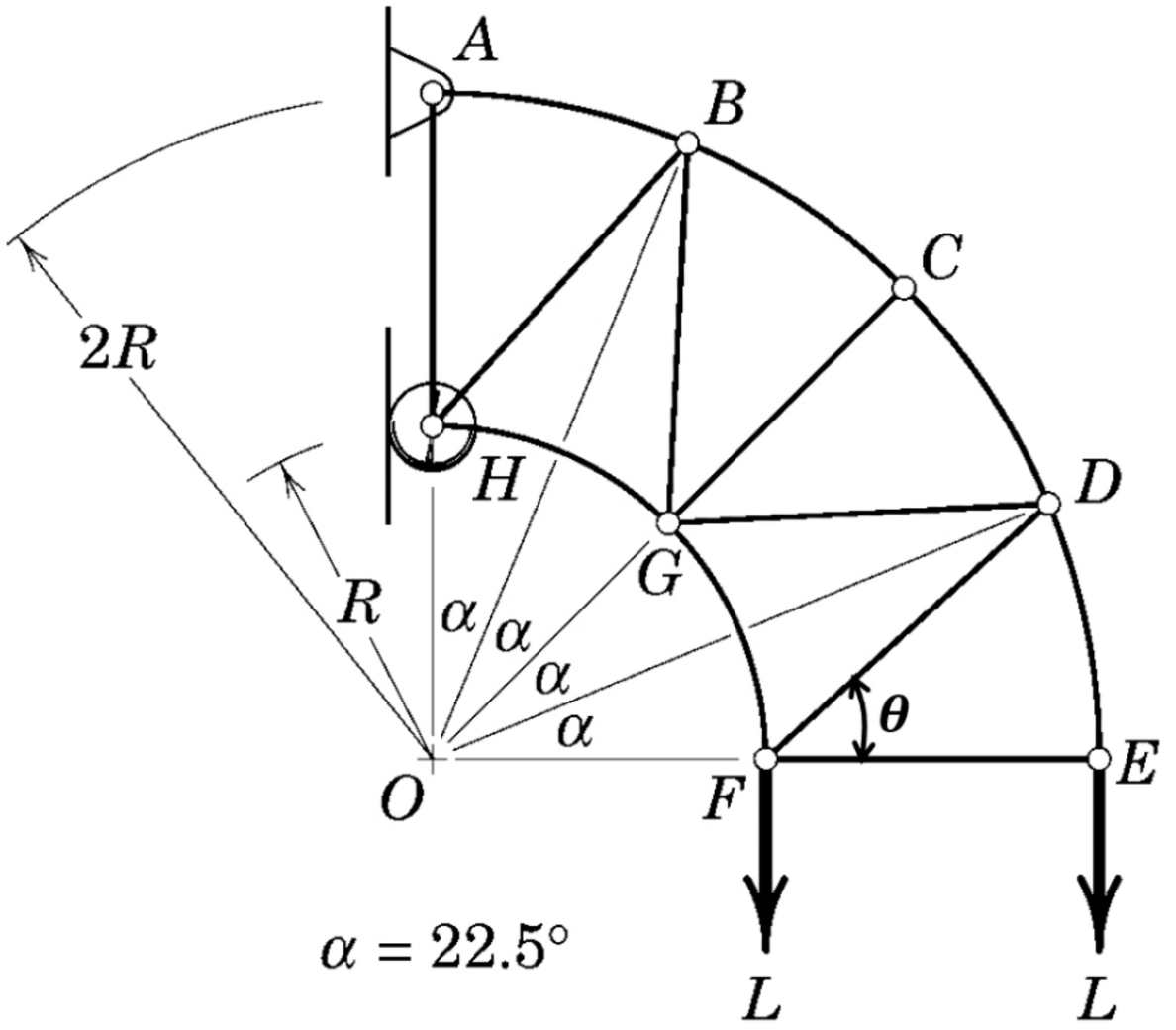
$\frac{\sqrt{x(x+80)}}{x+20} = \frac{2\left(\frac{x}{\sqrt{x(x+80)}}\right)}{1-\left(\frac{x}{\sqrt{x(x+80)}}\right)^2}$

$x = 20$

PLANE TRIGONOMETRY:

Example: For a given simple truss as shown, determine the angle formed by member DFE.

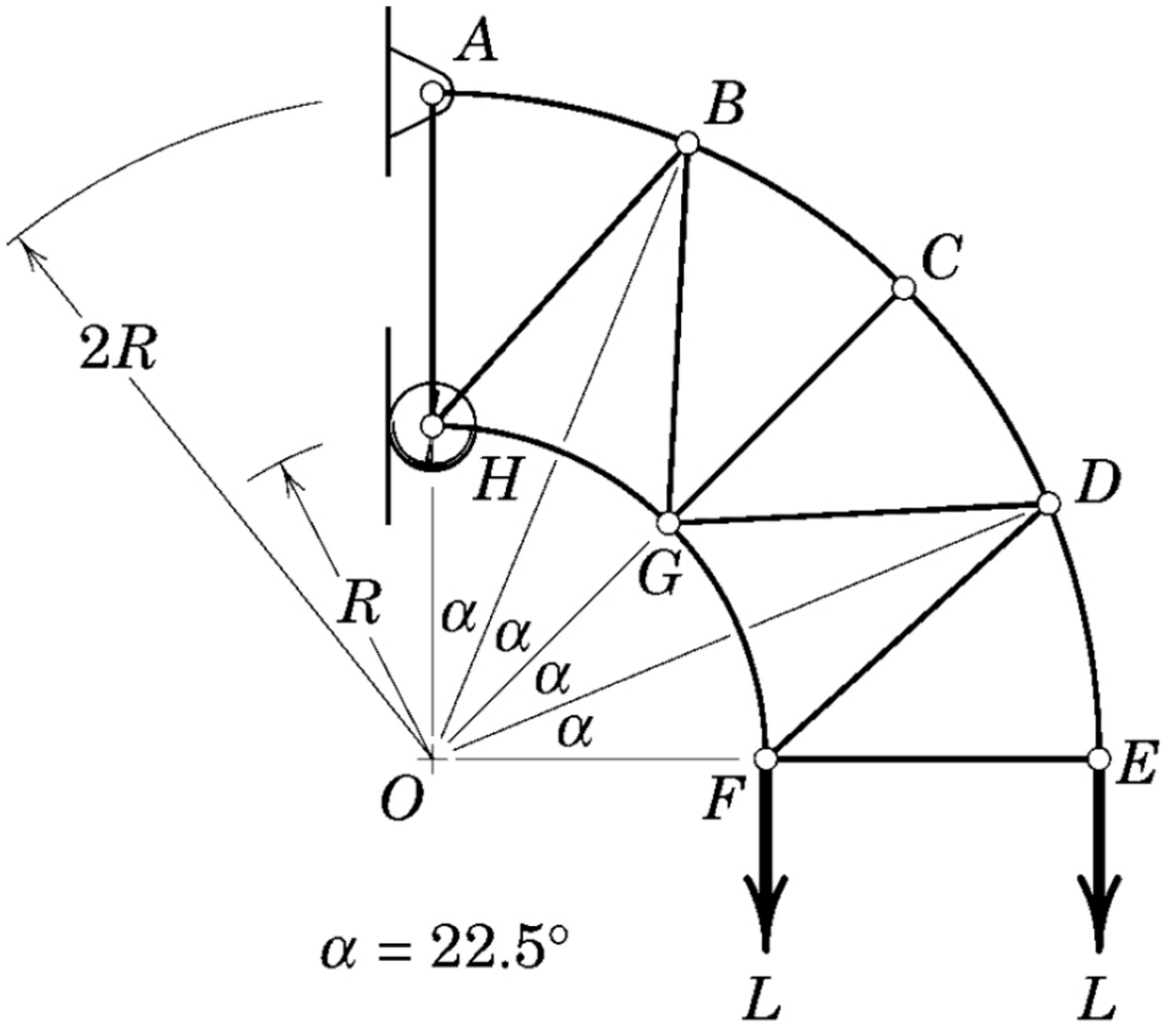
a) 42.1
a) 32.1
b) 42.1
c) 62.1



PLANE TRIGONOMETRY:

Example: For a given simple truss as shown, determine the angle formed by member DFE.

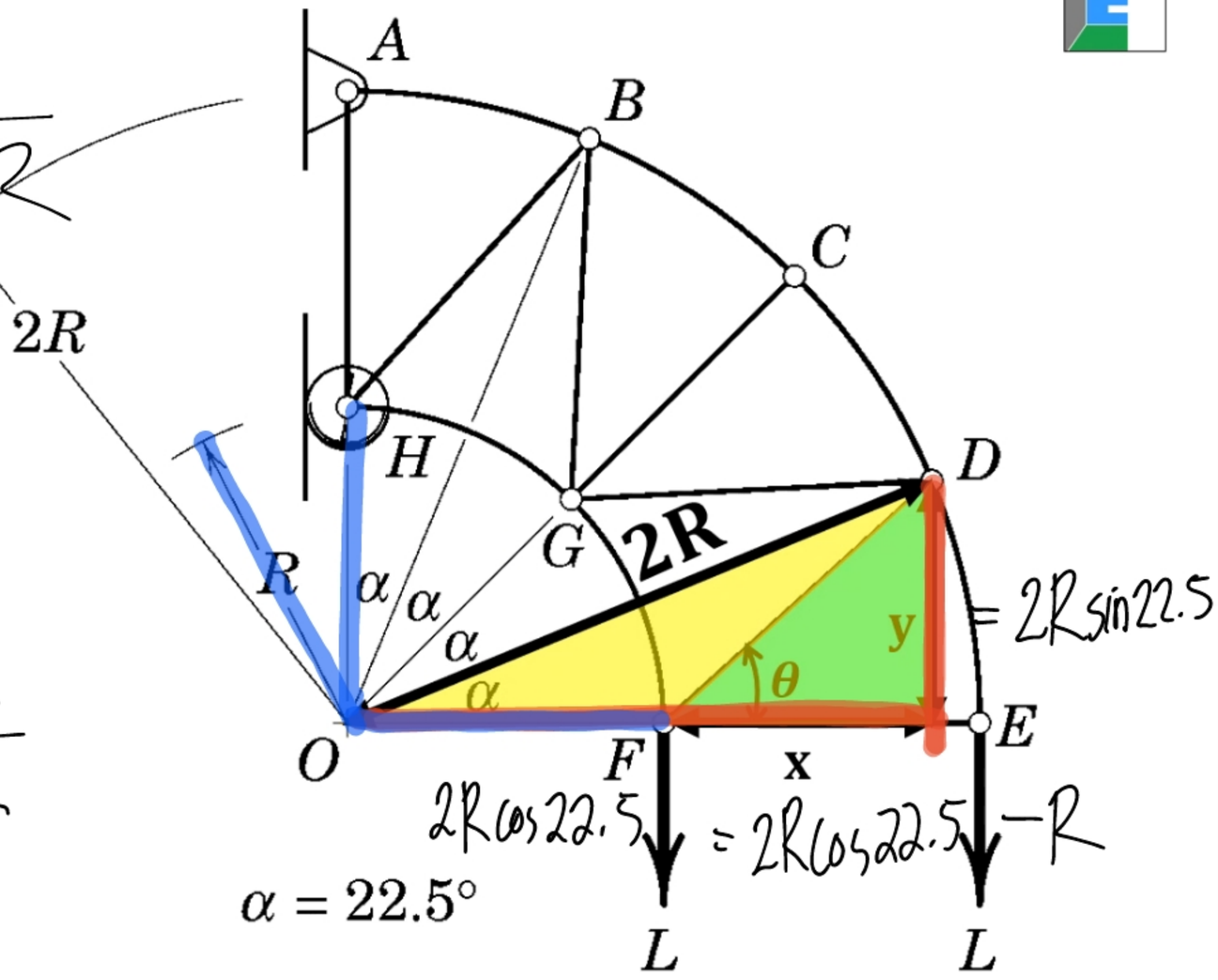
a) 42.1
b) 32.1
c) 52.1
d) 62.1



PLANE TRIGONOMETRY:

$$\tan\theta = \frac{y}{x} = \frac{2R\sin 22.5}{2R\cos 22.5 - R}$$

$$\tan\theta = \frac{R(2\sin 22.5)}{R(2\cos 22.5 - 1)}$$

$$\theta = 42.1°$$

$$\sin 22.5 = \frac{y}{2R}$$